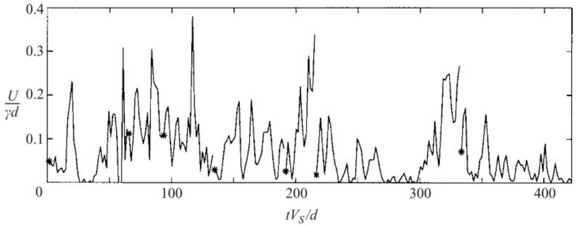
Mechanistic approach to bed load velocities

Kevin

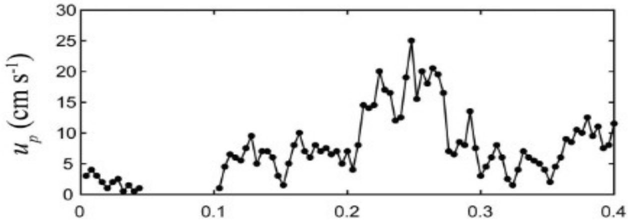
Description of bed load transport

Major problems:

1. Particle velocities

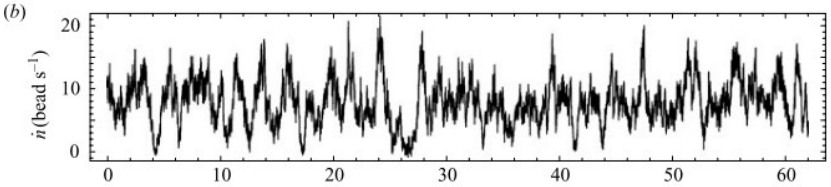


Charru et al (2004)

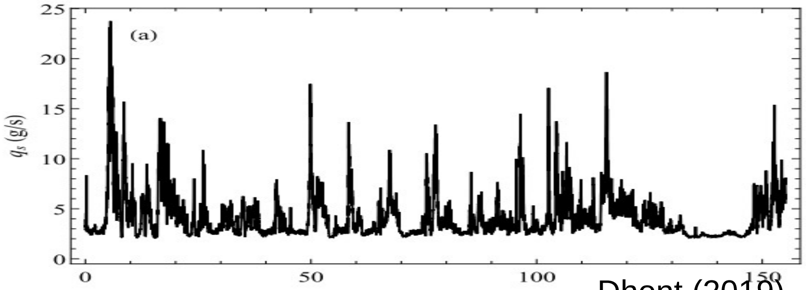


Roseberry et al (2012)

2. Sediment fluxes



Ancey et al 2008

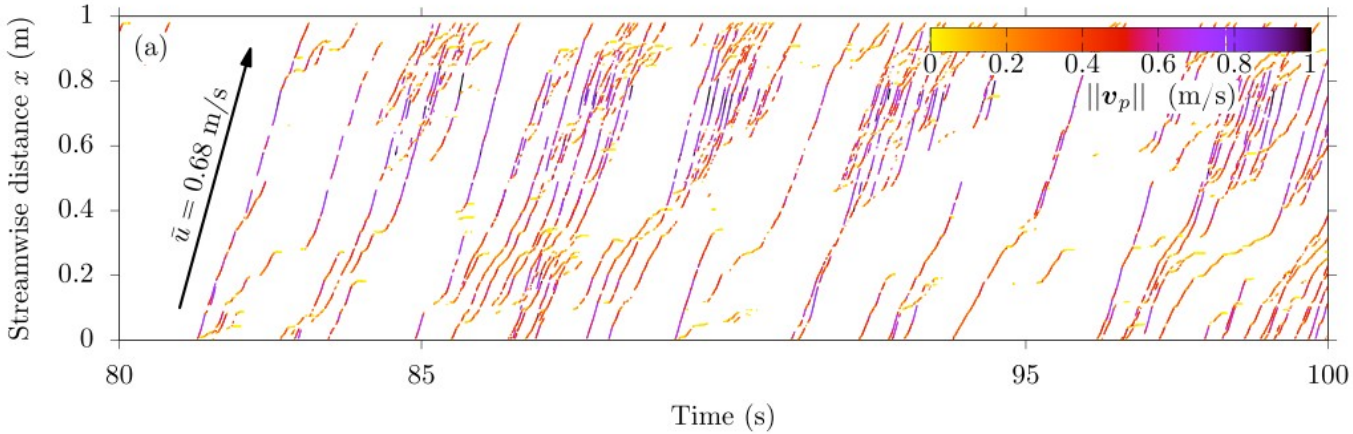


Dhont (2019)

Mean descriptions incomplete: How to obtain distributions?

Bed load velocities

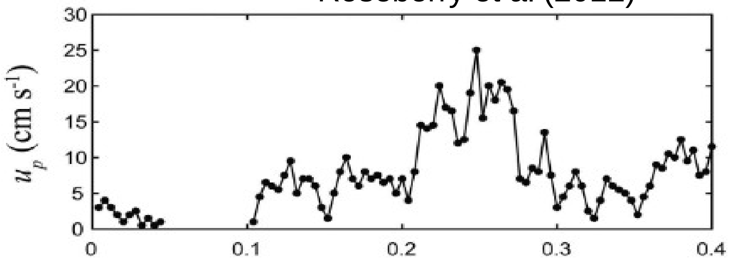
Heyman et al (2016)

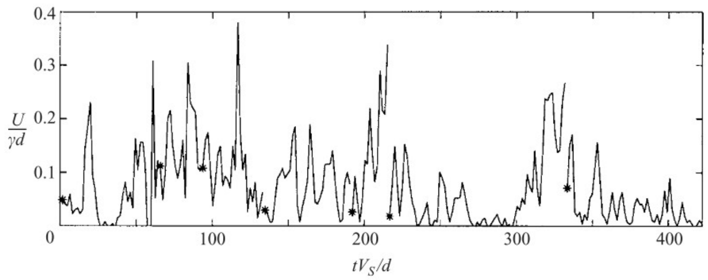


Vary with turbulence, particle-bed collisions, and more

Charru et al (2004)

Roseberry et al (2012)

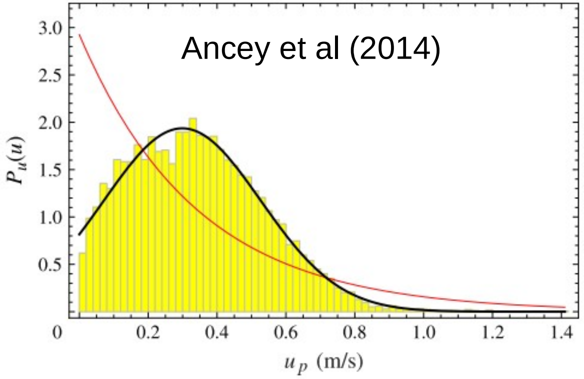


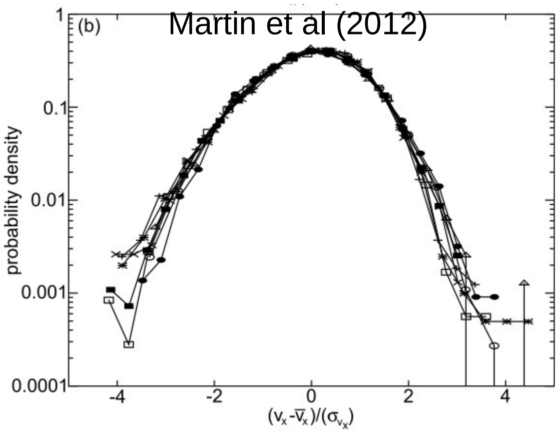


What is the velocity distribution like, and how to calculate it?

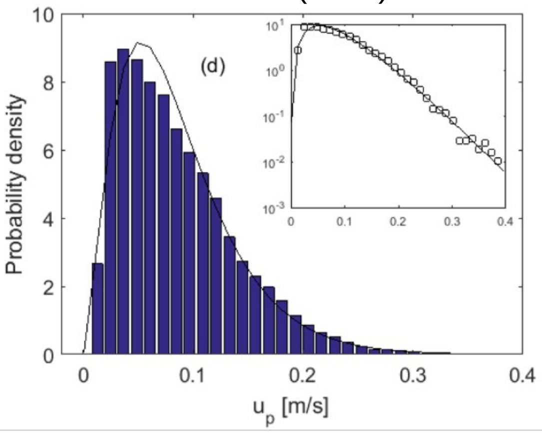
Major issue: are velocity distributions exponential or Gaussian?

Conflicting observations

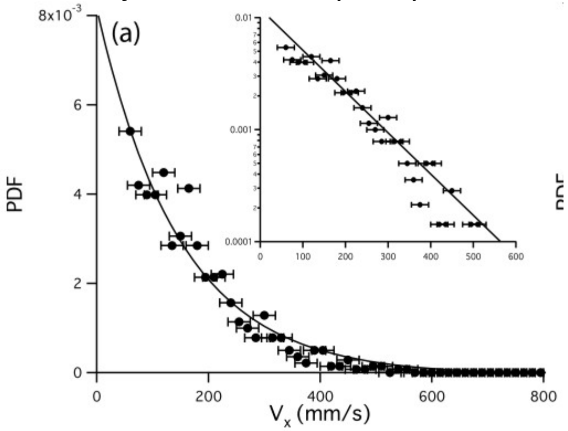




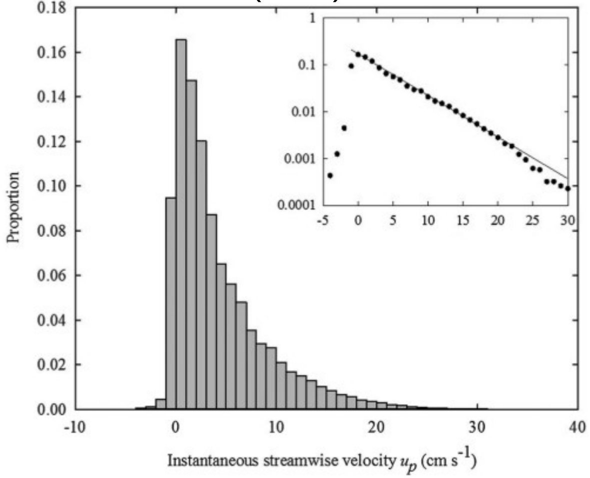
Liu et al (2019)



Lajeunesse et al (2010)



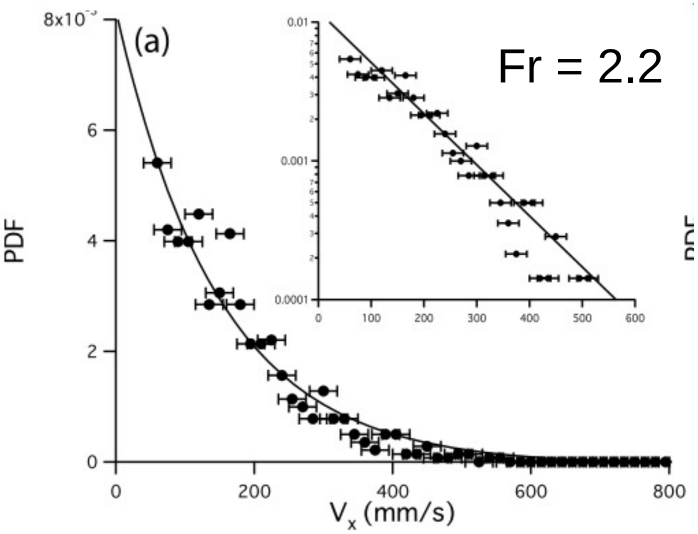
Fathel et al (2015)



Competing hypotheses for exponential vs Gaussian

1. Flow-controlled

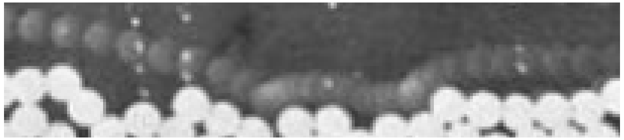
$$Fr = \frac{U}{\sqrt{gH}}$$

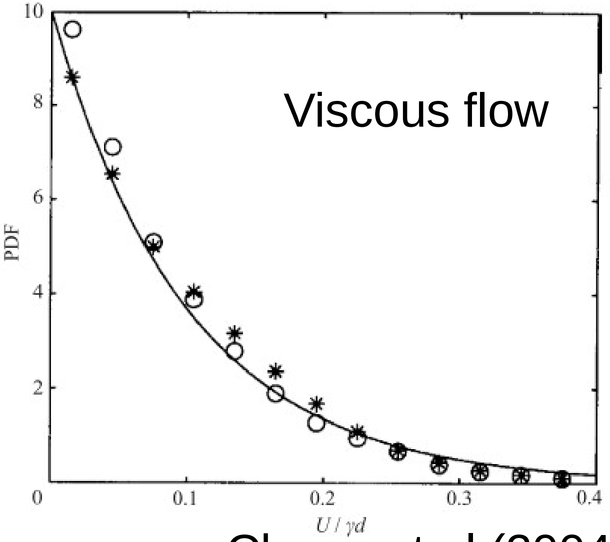


Lajeunesse et al (2010)

2. Collision-controlled

Gordon et al (1972)

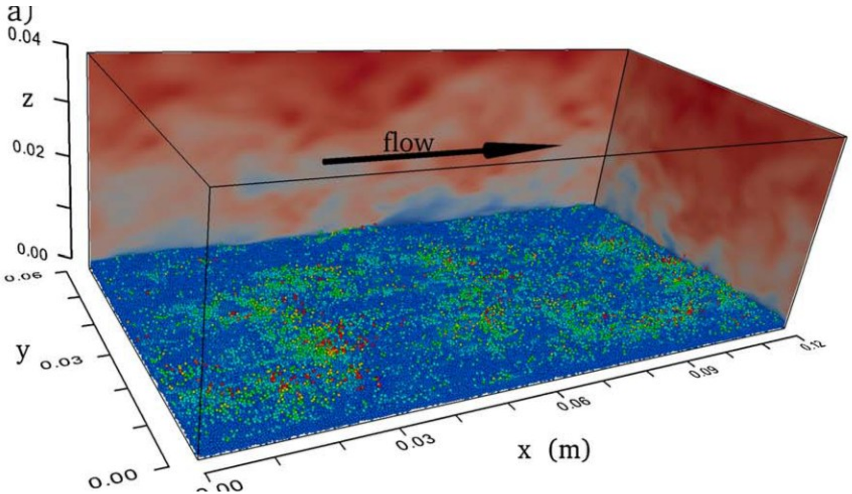




Charru et al (2004)

Modelling challenges: non-equilibrium & many degrees of freedom

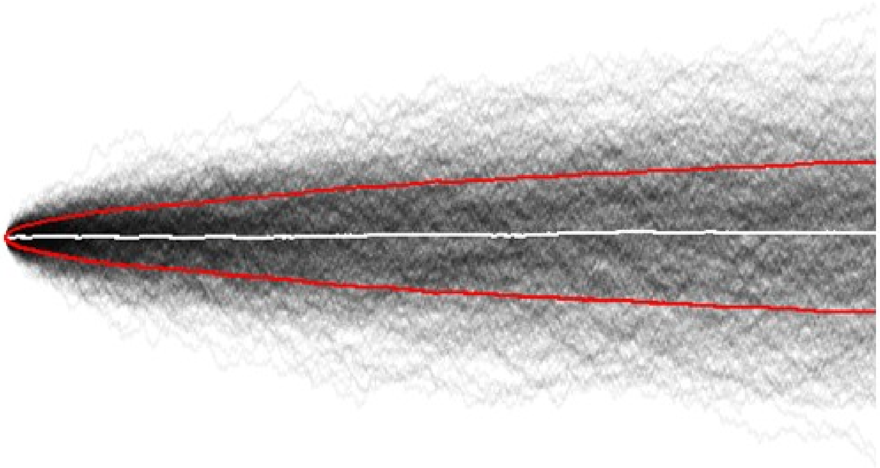
1. Computational physics:



Schmeeckle et al (2014)

(Account for all degrees of freedom)

2. Statistical mechanics:



(Average over “extra” d.o.f.)

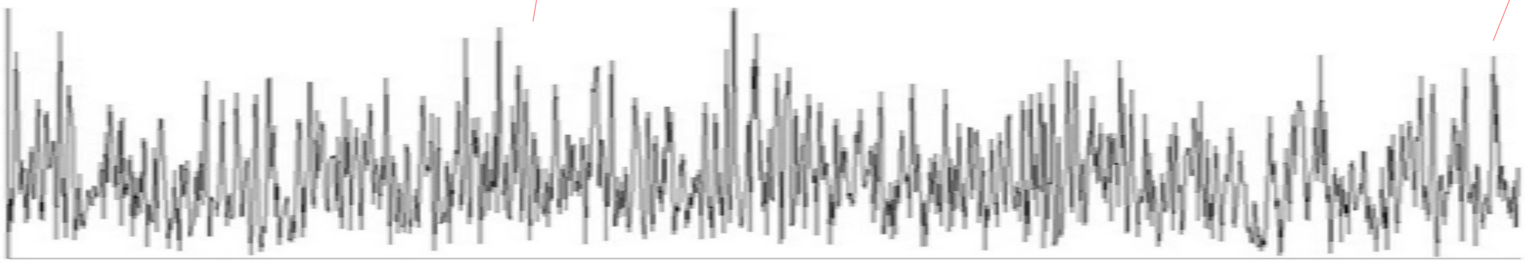
The statistical mechanics approach to bed load velocities

Ancey et al (2014)

$$t_r \frac{\mathrm{d}U}{\mathrm{d}t} = -(U - \bar{u}_s) + \sqrt{2D_u}\xi(t),$$

Fan et al (2014)

$$\frac{du_p}{dt} = -\Delta_x \cdot \mathrm{sign}(u_p) + F_x + \xi_x$$



Random driving terms
(Gaussian white noise)

The statistical physics approach to bed load velocities

Ancey et al (2014)

$$t_r \frac{\mathrm{d}U}{\mathrm{d}t} = -(U - \bar{u}_s) + \sqrt{2D_u}\xi(t),$$

Fan et al (2014)

$$\frac{du_p}{dt} = -\Delta_x \cdot \mathrm{sign}(u_p) + F_x + \xi_x$$

$$F = ma$$

Stochastic dynamics

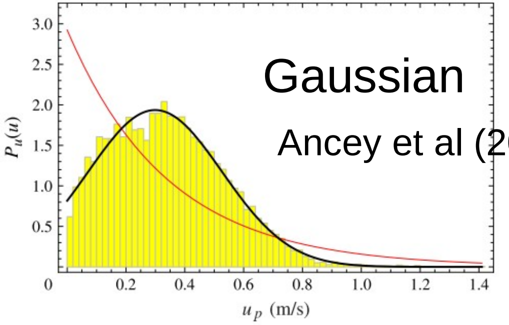
ensemble averaging

$$P(u)$$

Master equations/
velocity distributions

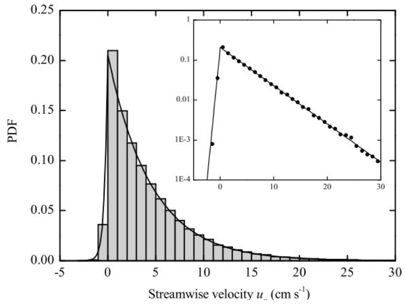
Major limitations of existing models:

1. Explain only part of the experimental results



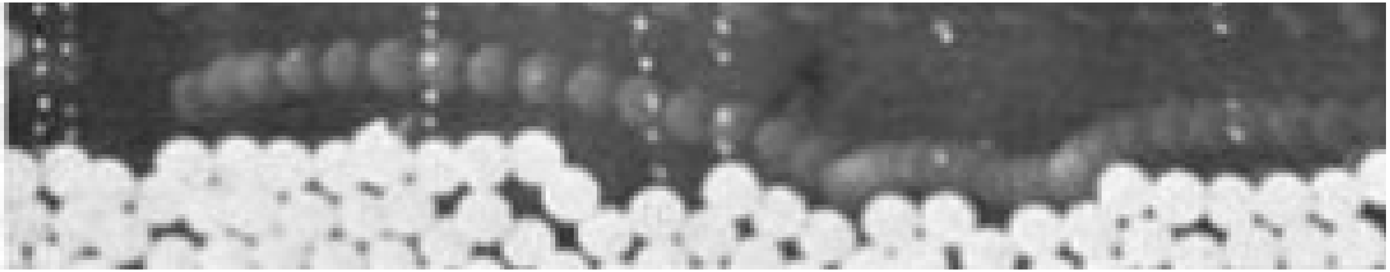
Gaussian

Ancey et al (2014)



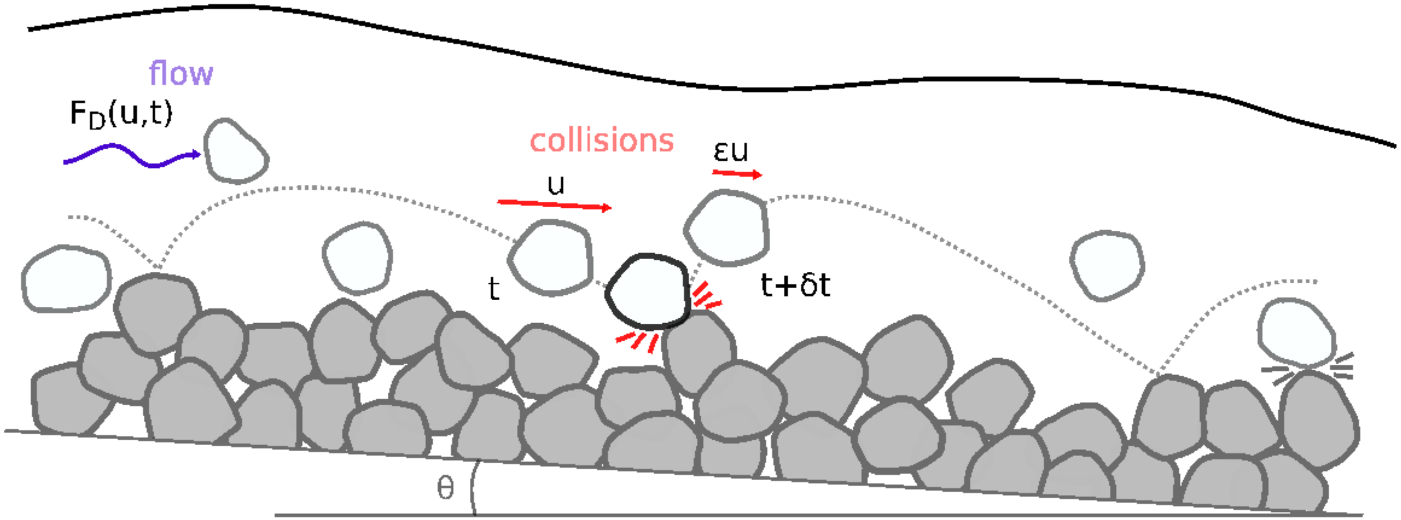
Exponential
– Fan et al (2014)

2. No incorporation of particle-bed collisions



Gordon et al (1972)

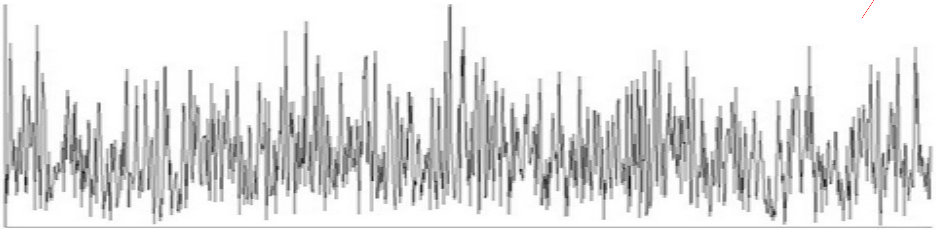
New development: “collisional Langevin approach for bed load velocities”



Objective: longitudinal velocities from fluctuating drag + episodic & dissipative collisions

Langevin equation with episodic collisions

$$m\dot{u}(t) = \Gamma + \sqrt{2D}\eta(t) - mu(t)\xi_{\nu,\varepsilon}(t),$$



Turbulent drag – Gaussian white noise

Collisions – a “Poisson jump process”

$$F_C(u,t) = -mu \sum_{k=1}^{N_\nu(t)} (1 - \varepsilon_k)\delta(t - \tau_k),$$

Collision forces – a random sequence of spikes

Average over both noises… Master equation for sediment velocity distribution:

$$\nu^{-1}\partial_t P(u,t) = -\tilde{\Gamma}\partial_u P(u,t) + \tilde{D}\partial_u^2 P(u,t) + I_c(u,t),$$

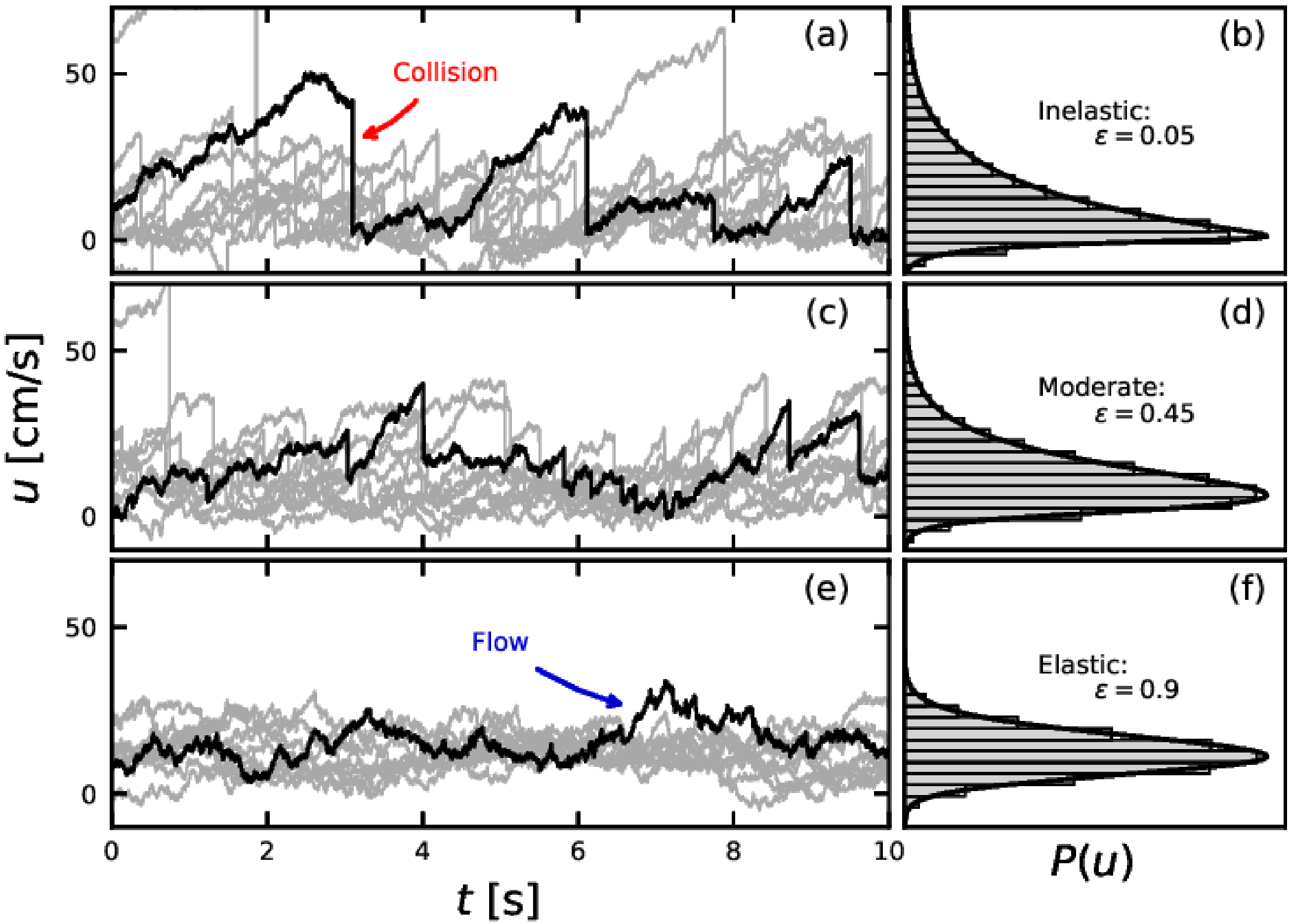
Advection + diffusion + nonlocal Boltzmann term

$$I_c(u) = -P(u,t) + \int_0^1 \frac{d\varepsilon}{\varepsilon} P\left(\frac{u}{\varepsilon},t\right)\rho(\varepsilon)$$

Distribution of restitution-like coefficients

(hi Sarah)

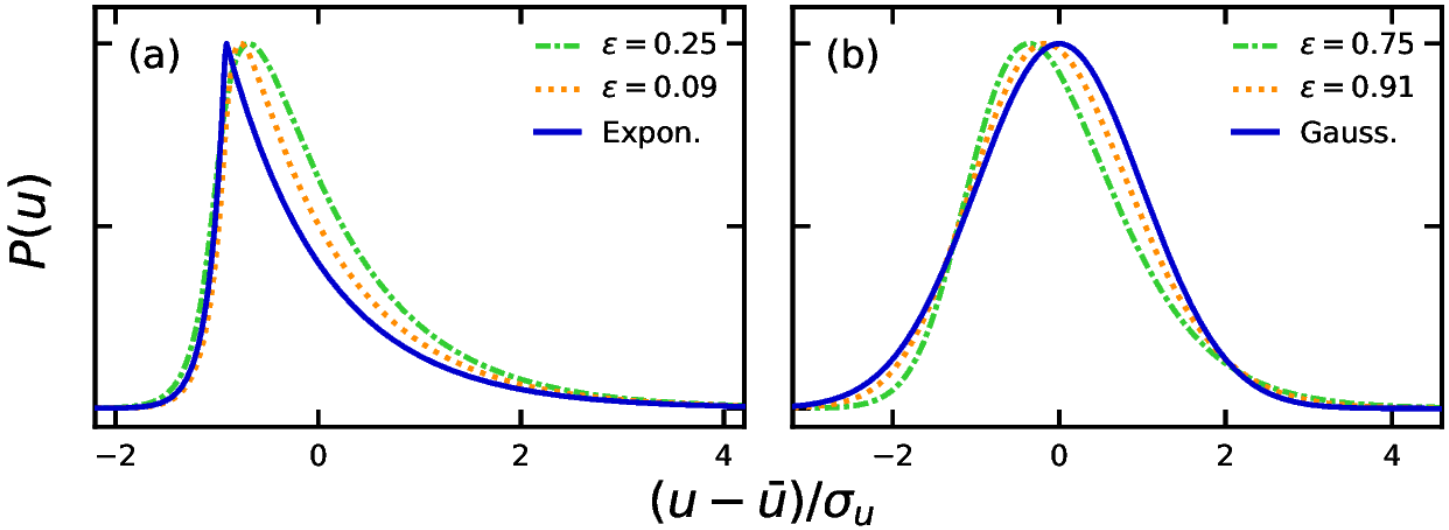
Analytical solutions for bed load velocities under turbulence and collisions



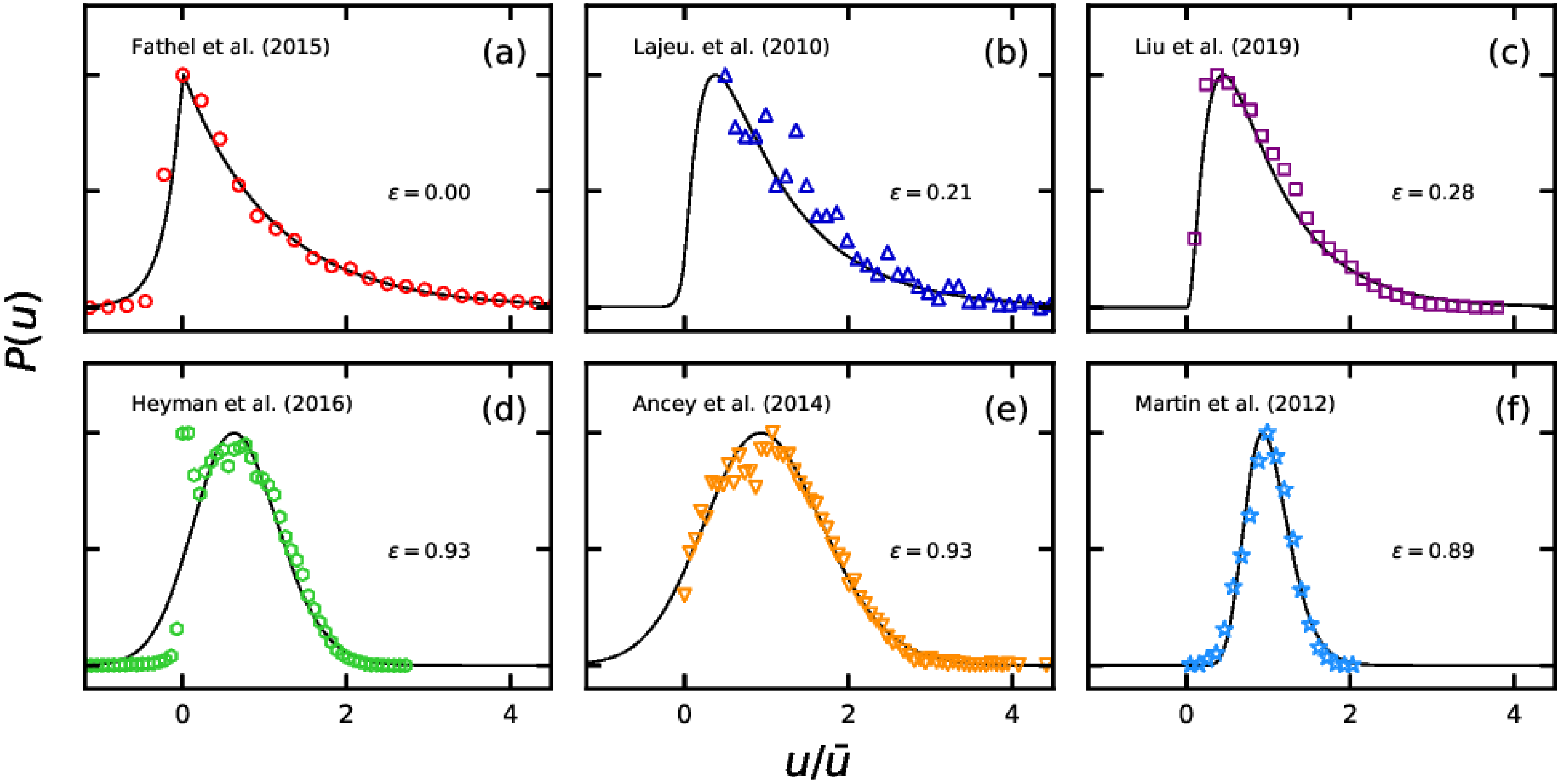
Velocity distribution exact solution (complicated)

$$P(u) = \frac{\theta(-u)}{K_+} \sum_{l=0}^{\infty} \frac{\varepsilon^{-l} e^{\lambda_+ \varepsilon^{-l} u}}{\prod_{m=1}^{l} (-d\lambda_+^2 \varepsilon^{-2m} + \gamma\lambda_+ \varepsilon^{-m} + 1)} + \frac{\theta(u)}{K_-} \sum_{l=0}^{\infty} \frac{\varepsilon^{-l} e^{\lambda_- \varepsilon^{-l} u}}{\prod_{m=1}^{l} (-d\lambda_-^2 \varepsilon^{-2m} + \gamma\lambda_- \varepsilon^{-m} + 1)},$$

Exact limits to Gaussian/exponential for fully elastic/inelastic collisions

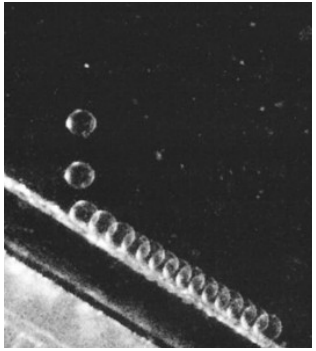


Experimental comparison:

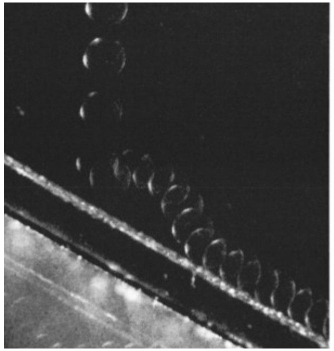


Episodic collisions allow fits of all experimental data (disclaimer..)

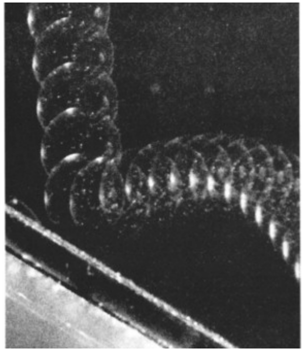
Context:..



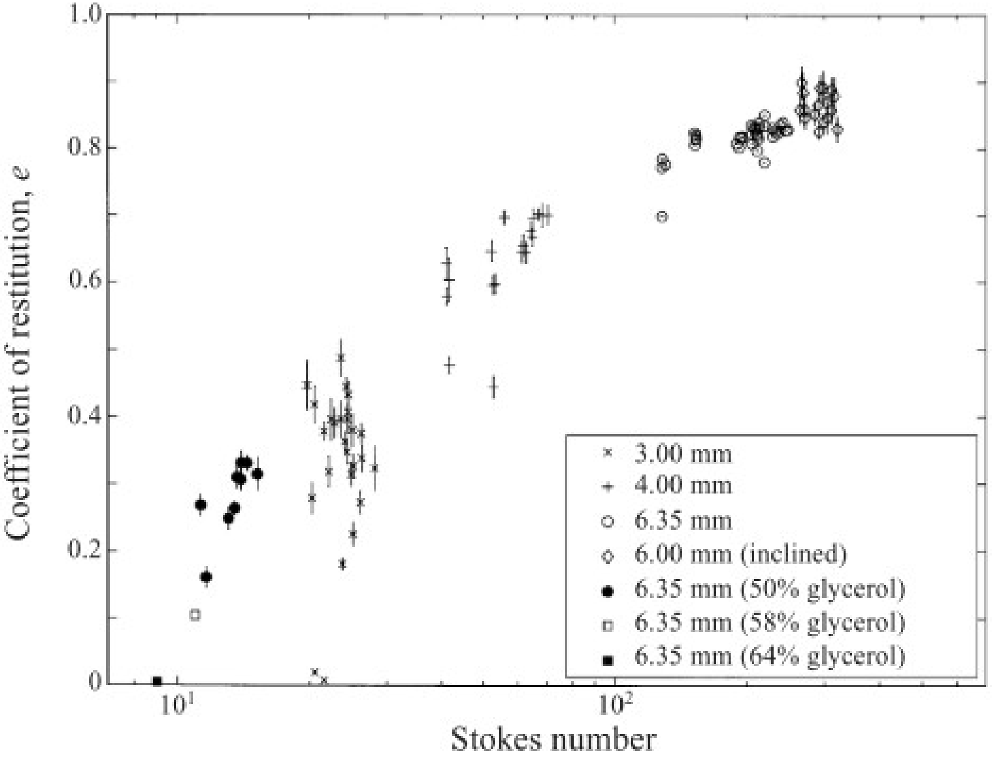
D = 1 mm



D = 2 mm



D = 5 mm



Collisions probably very important ….

Conclusions

1. New stochastic model with particle-bed collisions – exact solutions

$$m\dot{u}(t) = \Gamma + \sqrt{2D}\eta(t) - mu(t)\xi_{\nu,\varepsilon}(t),$$

2. Exponential / Gaussian conundrum explained by collision dynamics

Implications

Details of particle-bed collisions are important