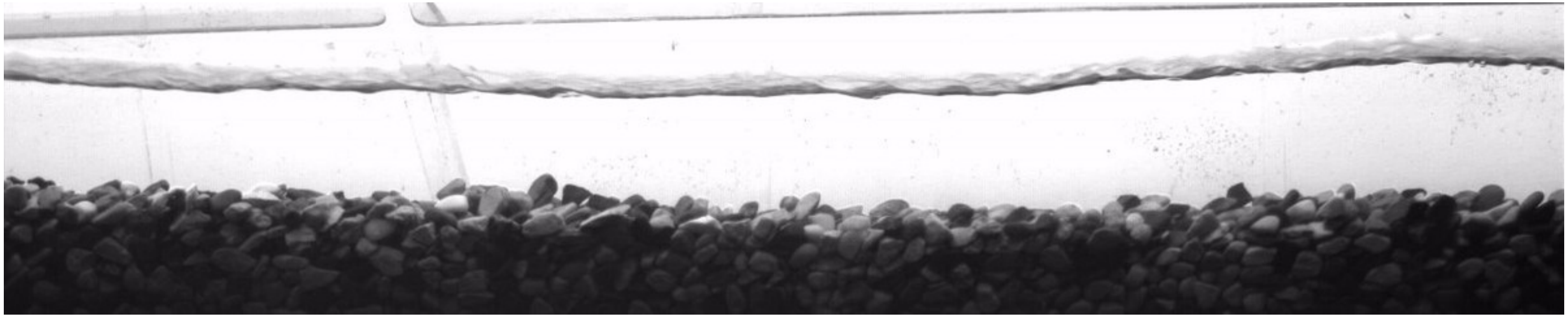
Mechanistic formulation of the bedload sediment flux

Kevin Pierce

Bedload transport



Major features:

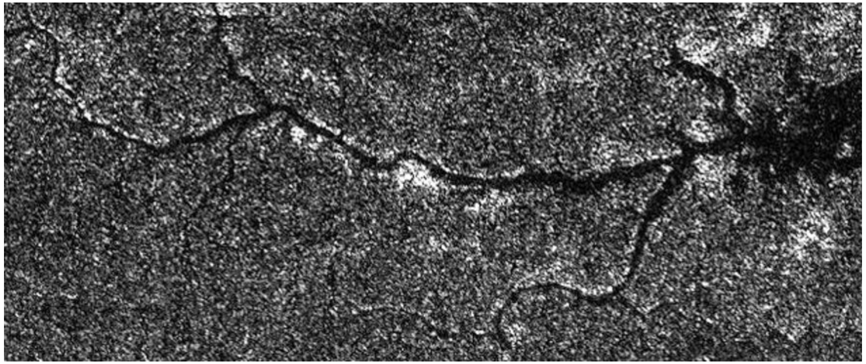
1. Motion-rest alternation
2. Velocity fluctuations among moving particles

Fundamentally, the flux originates from individual particles.

Significance of bedload transport

Geomorphology

Space science

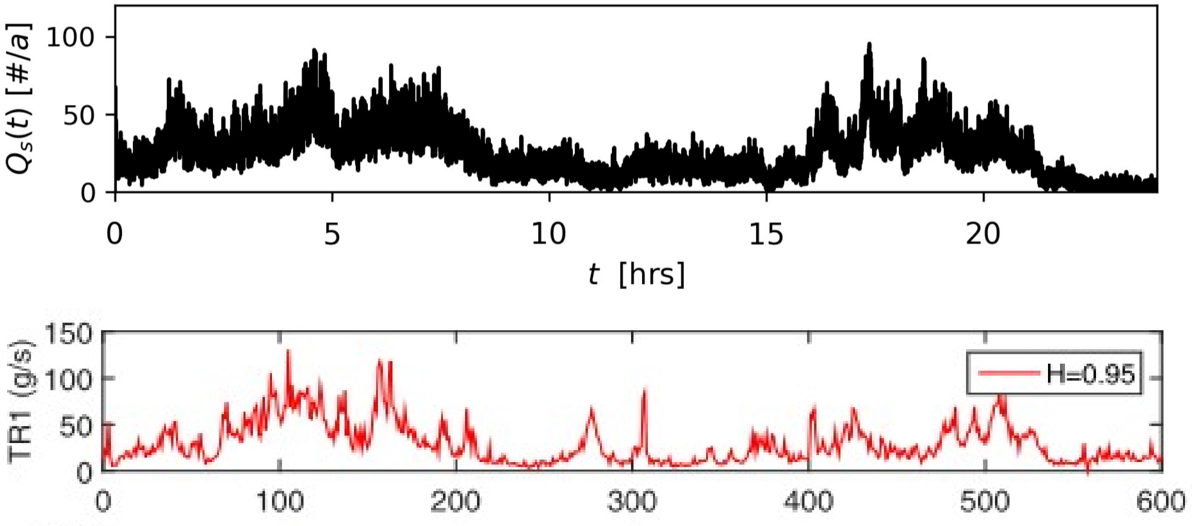


Hazard mitigation

Analogous processes

Characteristics of the bedload flux:

1. Transport rates ***fluctuate***

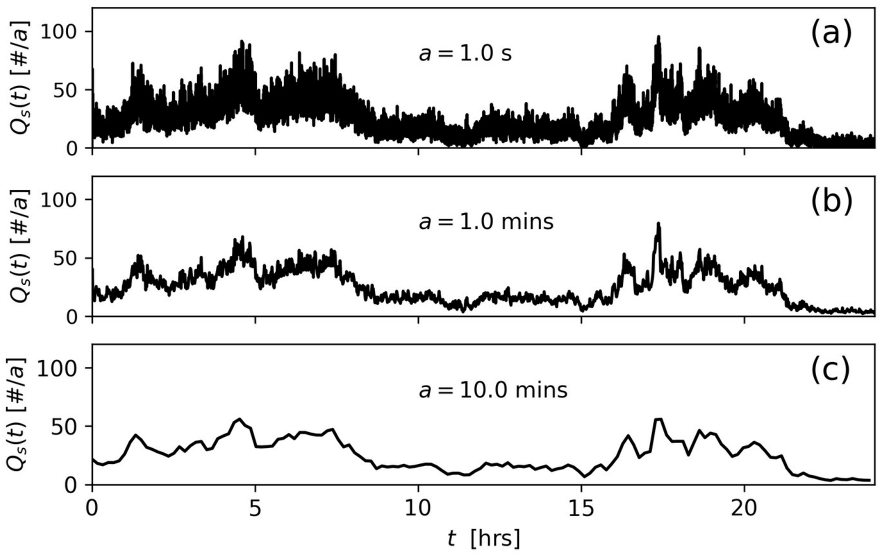


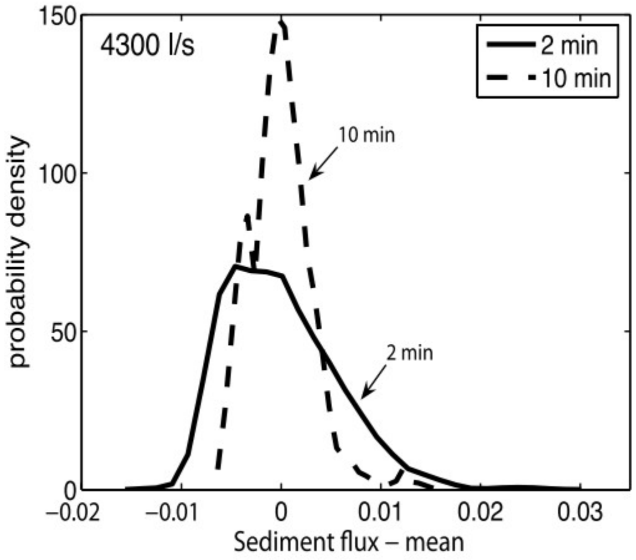
How big are bedload fluctuations?

How are these related to grain-scale processes?

Characteristics of the bedload flux

2. Because of fluctuations, measurements always involve ***averaging***

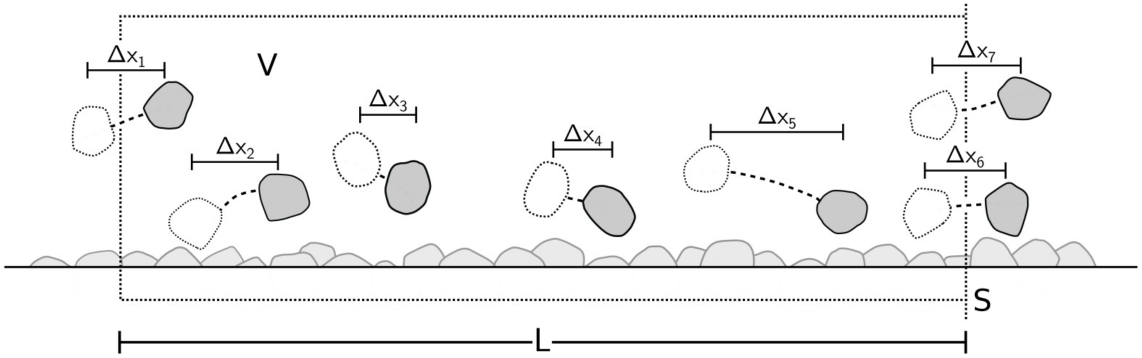




How does the flux depend on the averaging scale?

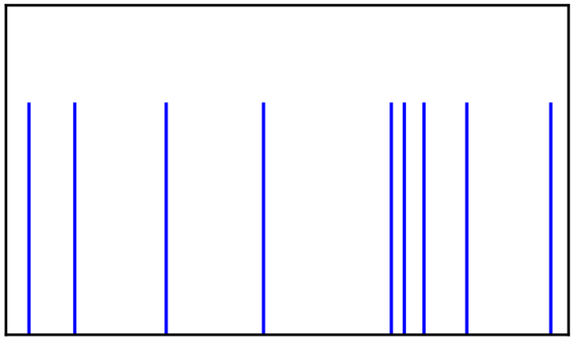
Predictions should include (1) **fluctuations** and (2) **dependence on averaging scales.**

Earlier approach to fluctuations & scale dependence



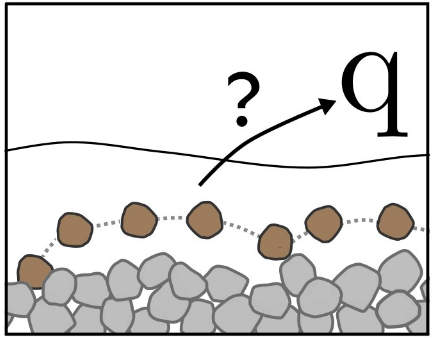
Renewal theory: Particles arrive randomly – What's their arrival rate?

$$q(T) = \frac{\mathcal{N}(T)}{T}$$



Heuristic – not based on particle dynamics. Is it possible to make it mechanistic?

Problem:

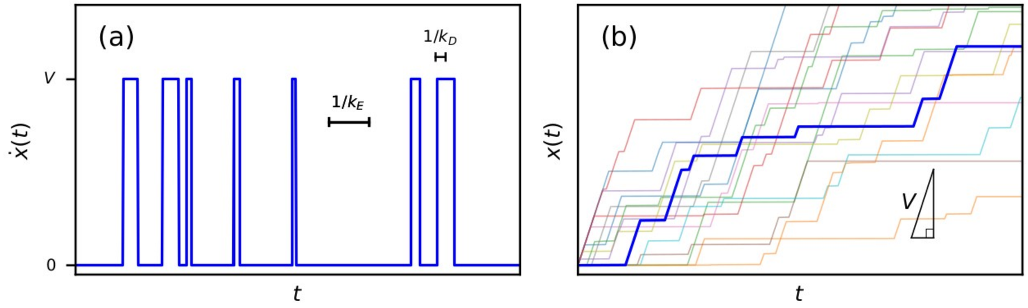


Q: Can we calculate the flux directly from particle motions?

(with observation scales and fluctuations)

Big prerequisite problem:

Existing models of particle transport are too crude to use for this.



Constant motion velocity.

[Example of Lajeunesse et al (2017)]

pQ: Can we (first) develop a realistic-enough model of particle transport?

pQ: How to describe grain-scale transport?

Main features (1) **variable velocities** during motions (2) **motion-rest alternation**

Relevant early work:

$$\dot{x}(t) = v(t)$$

$$\dot{v}(t) = F(u) + \xi(t)$$

Physica VII, no 4 April 1940

BROWNIAN MOTION IN A FIELD OF FORCE AND THE DIFFUSION MODEL OF CHEMICAL REACTIONS

by H. A. KRAMERS

Leiden

Kramers (1940) described particle movement in a turbulent force field

$$\partial_t W(x, v, t) = [-v\partial_x + \partial_v\{-F(v) + \Gamma\partial_v\}]W(x, v, t)$$

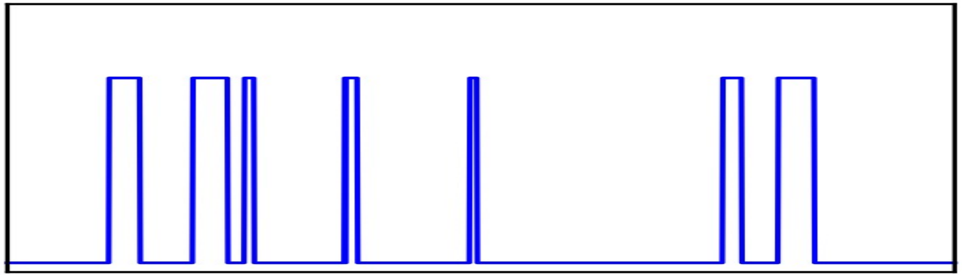
pQ: An "Intermittent Kramers Equation" for bedload:

$$\dot{x}(t) = v(t)\sigma(t),$$

$$\dot{v}(t) = [F(u) + \xi(t)]\sigma(t)$$

Equations of motion

Turbulent forces get *switched on and off*



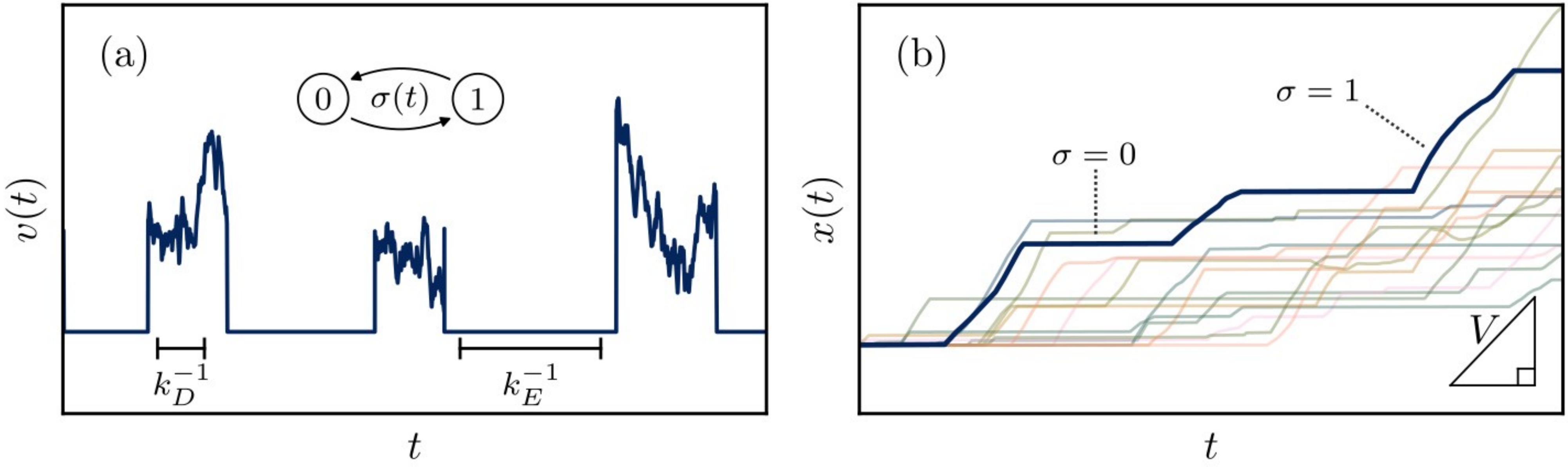
After some work, imitating Kramers:

$$\partial_t(\partial_t + k)W(x,v,t) = (\partial_t + k_E)[-v\partial_x + \partial_v\{-F(v) + \Gamma\partial_v\}]W(x,v,t)$$

Particle dynamics in a turbulent force field ***with motion-rest alternation***

pQ: A more realistic description of bedload transport:

Particles entrain and deposit; their movement velocities fluctuate.



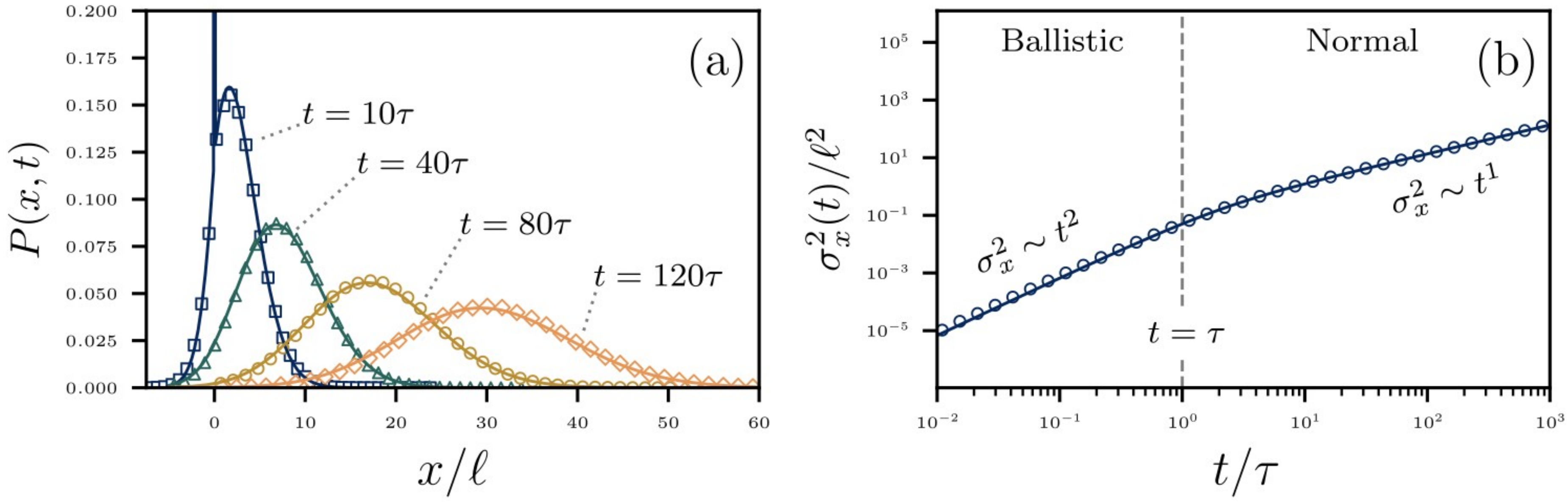
The approach is ***mechanistic*** – all described by F=ma

pQ: Approximate solution of particle displacement

Assuming particles accelerate rapidly after entrainment, IKE becomes:

$$\left[\partial_t^2 + k\partial_t + V\partial_x\partial_t + k_E V\partial_x - D\partial_x^2\partial_t - k_E D\partial_x^2\right] P(x,t) = 0$$

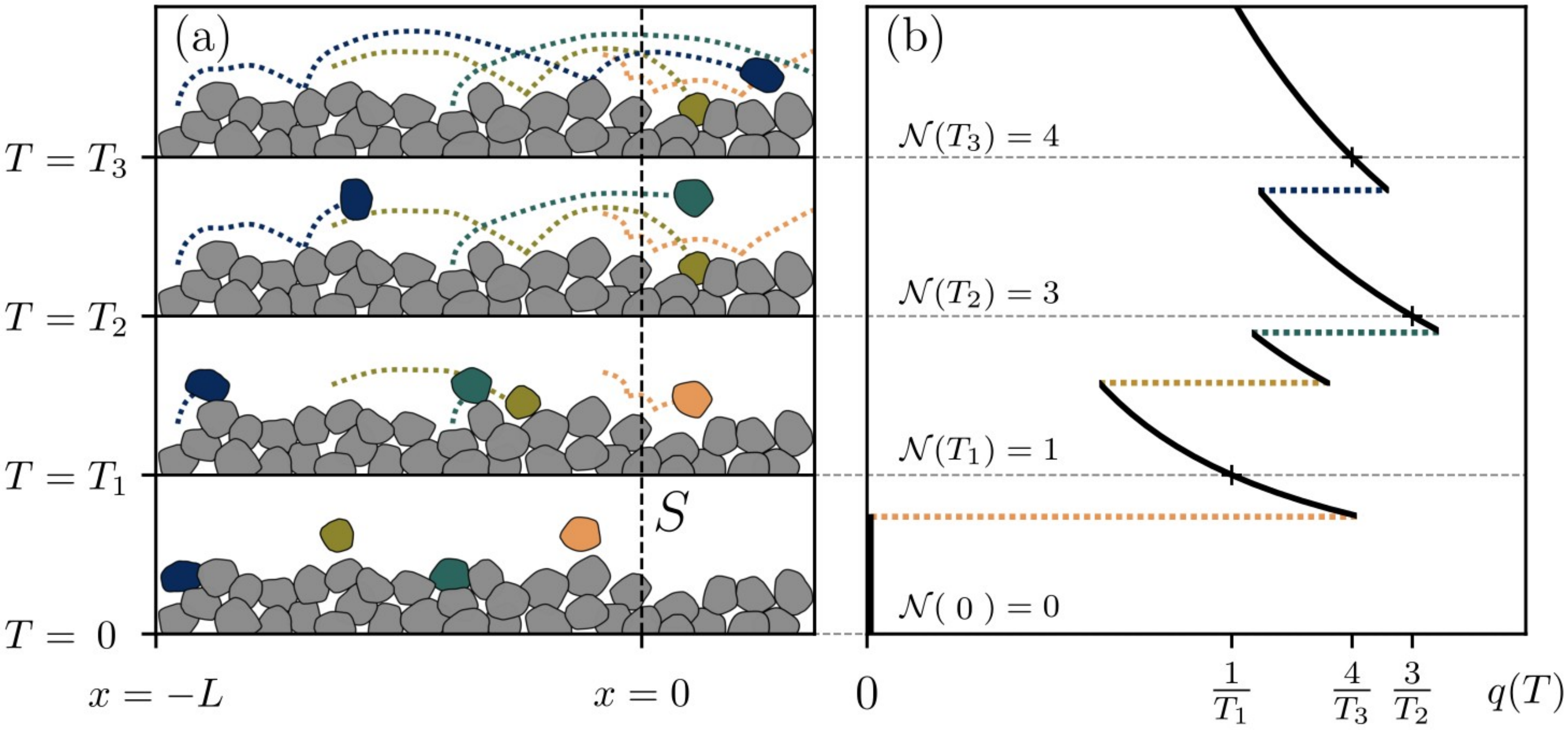
This equation can be exactly solved, giving:



So we have a vastly-improved model of particle displacement. pQ=solved

Q: How can we formulate the flux from grain-scale motions?

Conceptually, count the rate of particles crossing a control surface



Q: Probability distribution of the flux from mechanistic theory

Mathematically, rate of particles crossing control surface in observation time *T*

$$q(T) = \frac{1}{T}\sum_{i=1}^{N} I_i(T)$$

After probability stuff:

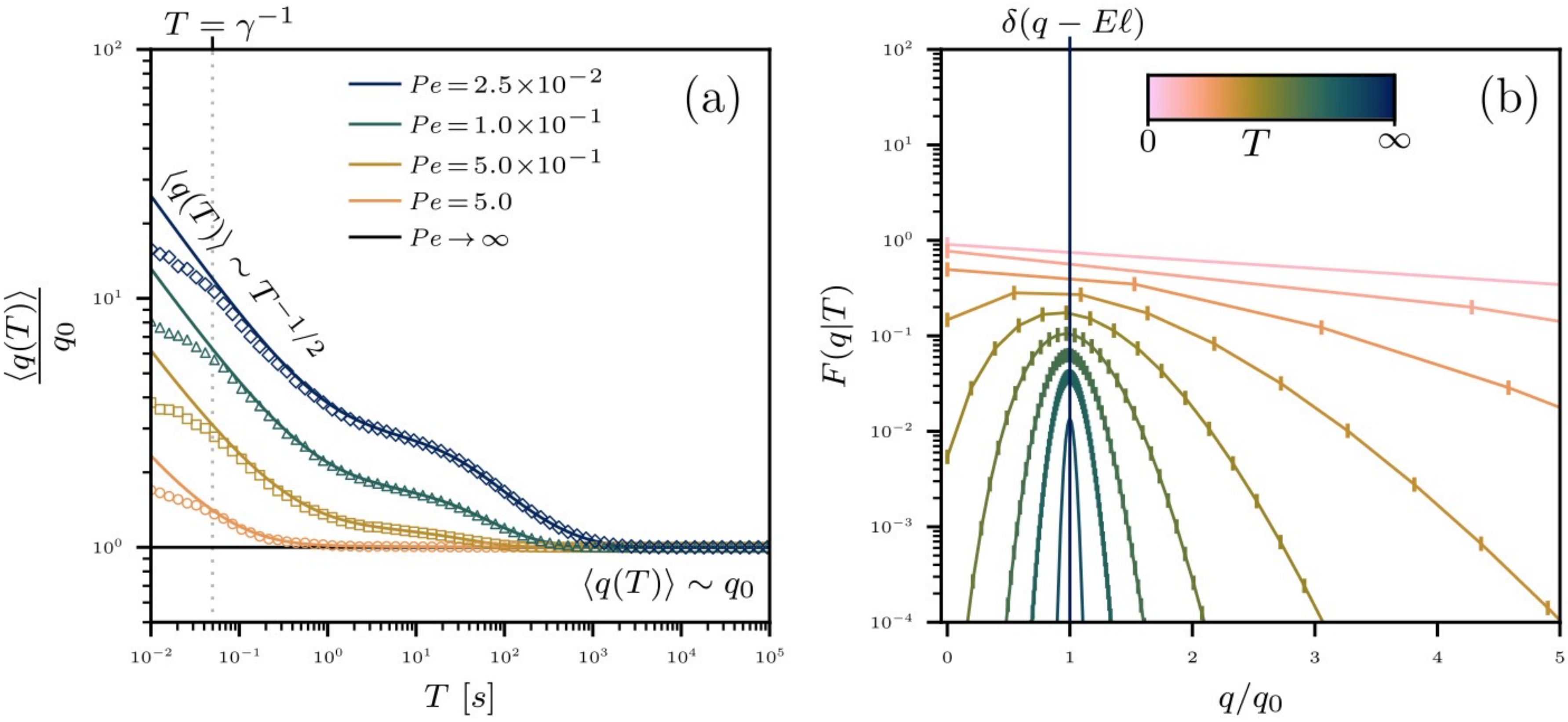
$$F(q|T) = \sum_{n=0}^{\infty} \frac{\Lambda(T)^n}{n!} e^{-\Lambda(T)} \delta\left(q - \frac{n}{T}\right)$$

The probability distribution of the flux ***conditional on T***

Q = solved

Q: Probability distribution of the flux from mechanistic theory

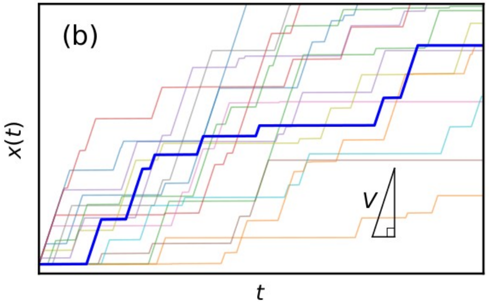
Transport rates fluctuate and depend on observation time

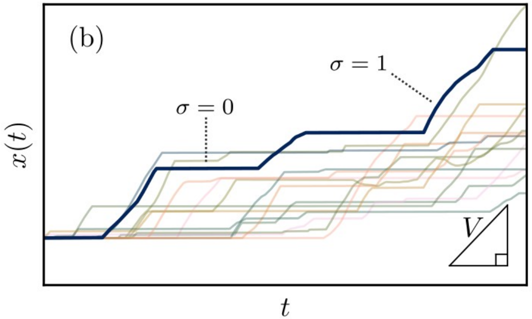


Flux is deterministic only at infinite observation times.

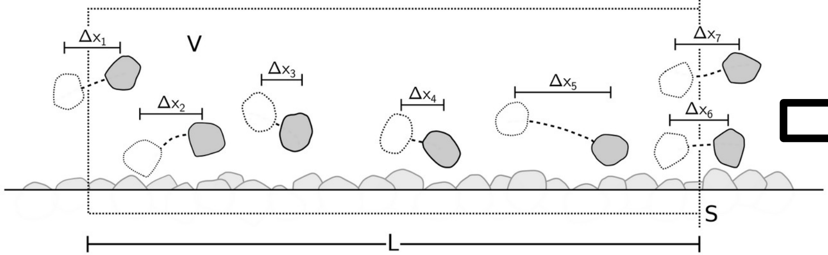
Summary of results:

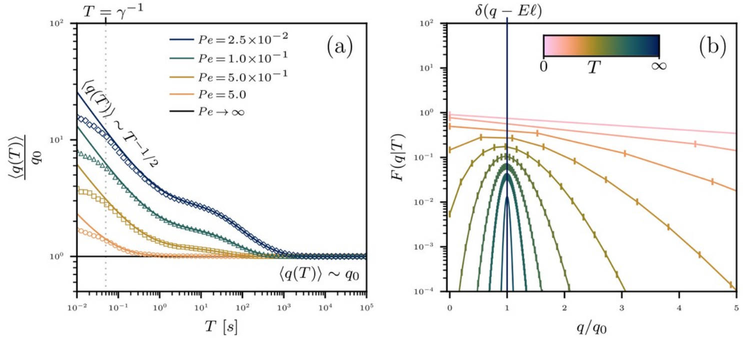
Improved earlier particle-scale transport models (pQ)





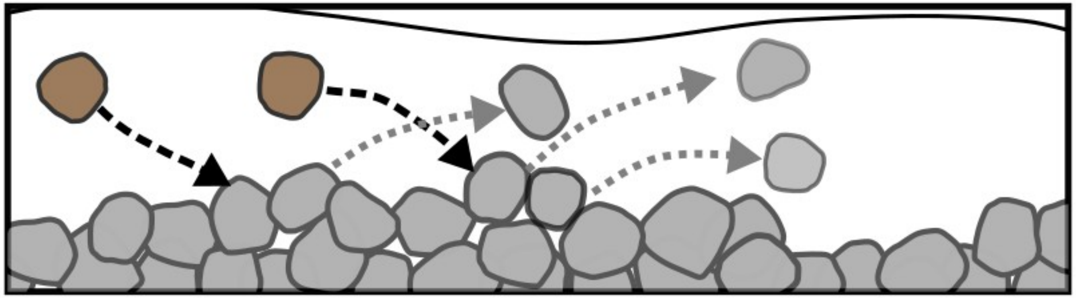
Calculated the flux and its scale dependence with a mechanistic model (Q)





A next step:

Interactions between grains (i.e., collisions -- wider fluctuations)



Since everything is Newtonian, the governing equations can be written down:

$$\dot{x}_i(t) = v_i(t)\sigma_i(t)$$

$$\dot{v}_i(t) = [F_i(v_i) + \sum_{j \neq i} G_{ij}(x_i, v_i, t, \{x_j, v_j\}) + \xi(t)]\sigma_i(t)$$

Solving them though . . .

Conclusion:

1. Described particle displacements with a mechanistic approach

2. Used this formulation of displacement to calculate the flux

3. Flux adopts inherent variability from grain-scale motions

4. The amount of variability depends on the observation time

5. This variability goes away at infinite observation times

Editing . . .

Mechanistic description of the bedload sediment flux

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Thanks!