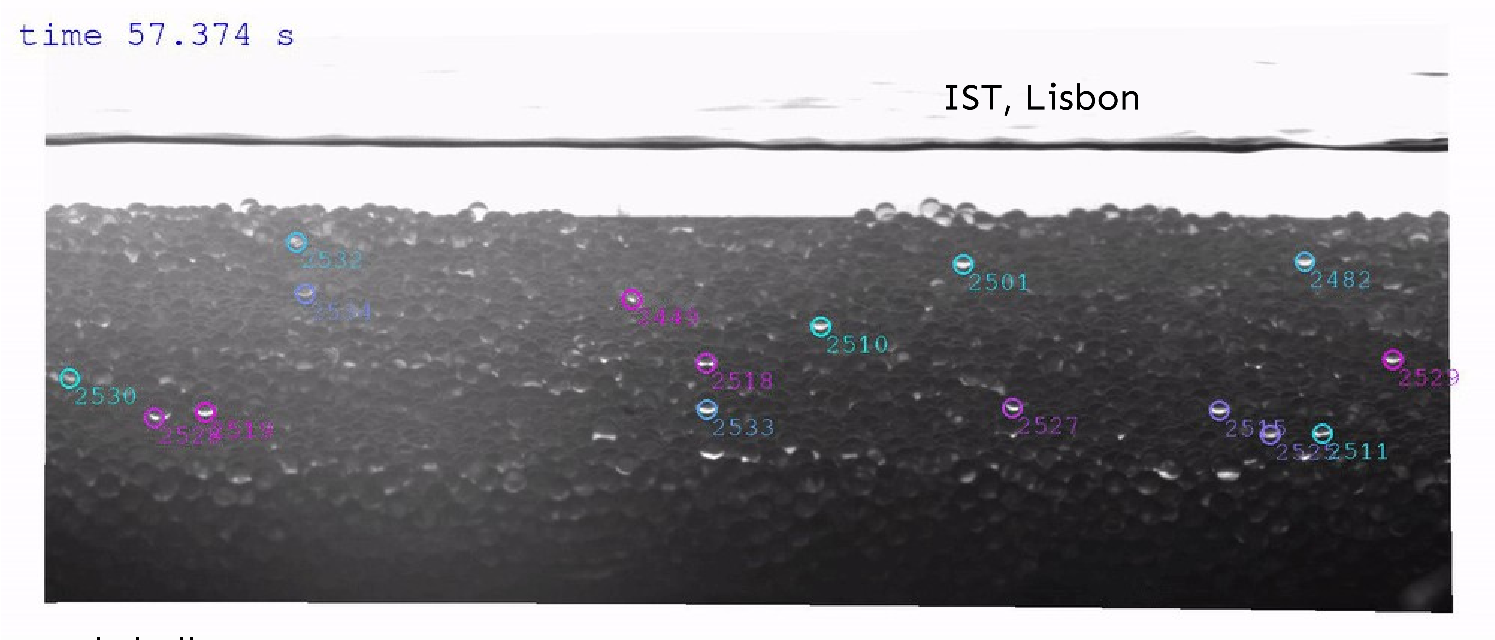
The stochastic movements of individual stream-bed grains

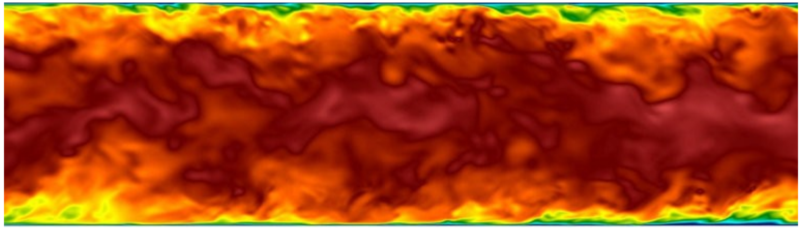
PhD defence, 31 Aug 2021

Kevin Pierce



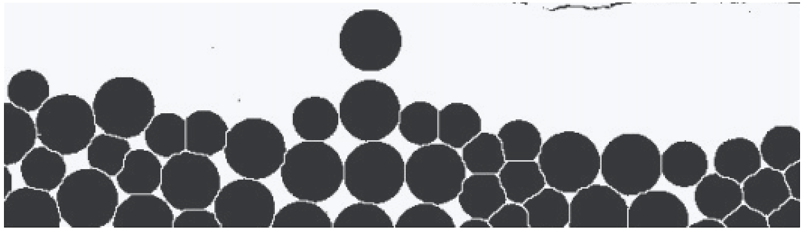
Fundamental challenges:

1) Turbulence



Dorschner et al (2016)

2) Granular surfaces

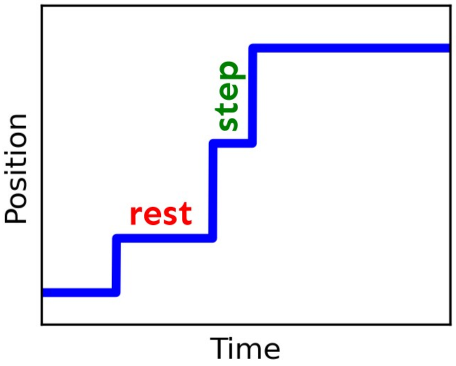


Martin (2013)

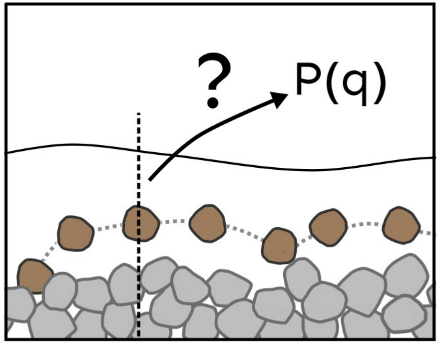
Probabilistic descriptions are suggested

Ch 1. Introduction – Limitations of existing probabilistic approaches

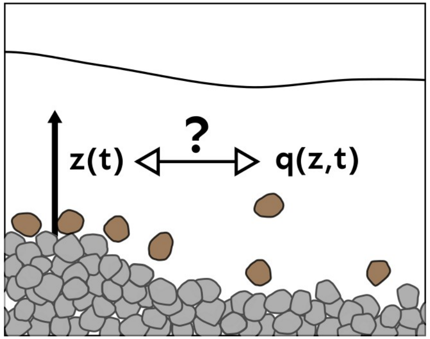
Problems of the thesis:



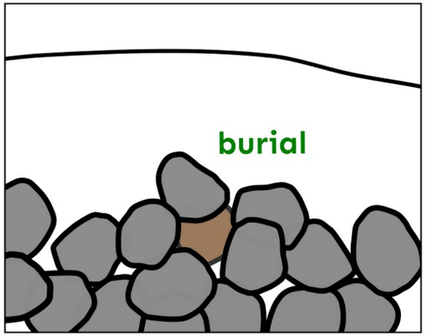
(1) Oversimplified trajectories



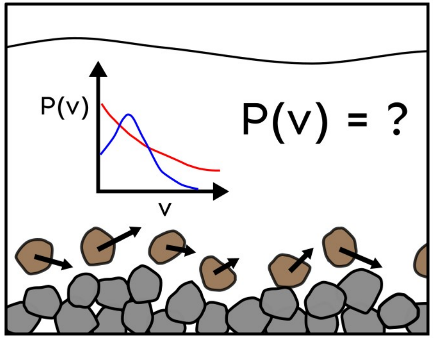
2. Incomplete linkage between particle trajectories and flux



3. Influence of bed elevations is unclear



4. Sediment burial has not been accounted for

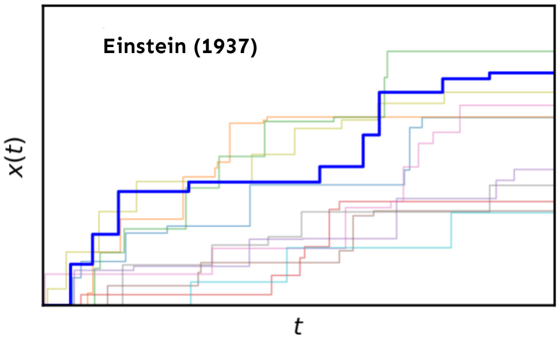


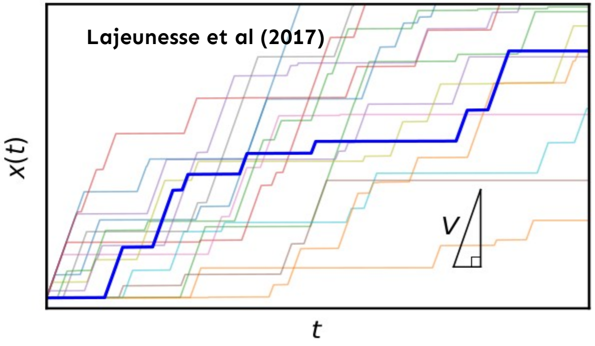
5. Incomplete understanding of particle movement velocities

Ch 2. Mechanistic-stochastic formulation of the bedload flux probability distribution

Problem statement:

Existing particle trajectory models are oversimplified





Progress has been extremely slow – clear problems but technical issues

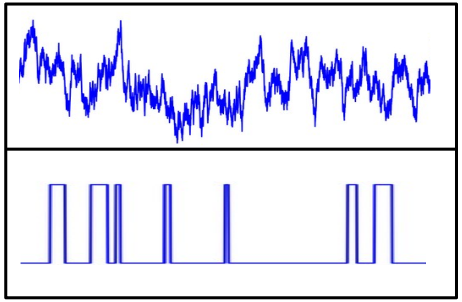
Objectives:

(a) Improve particle trajectory description to include velocity fluctuations

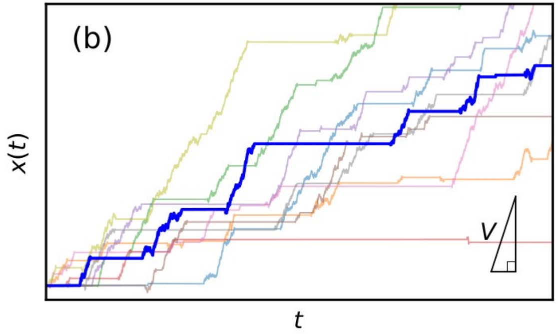
(b) Apply this to derive the sediment flux probability distribution

Ch 2. Mechanistic-stochastic formulation of the bedload flux probability distribution (pdf)

Formulation:

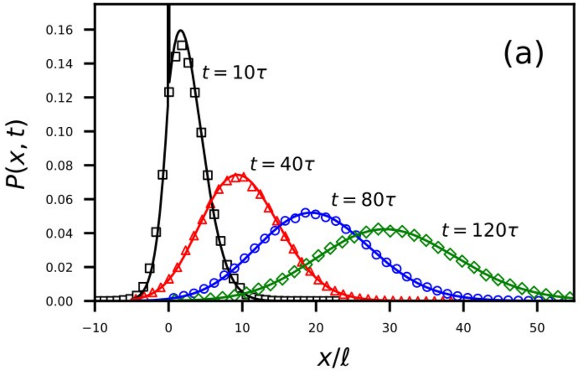


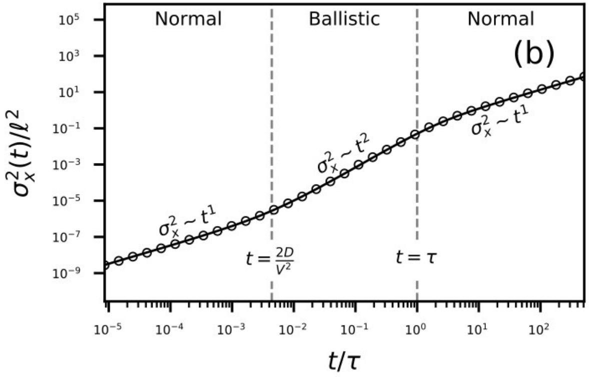
$$\dot{x}(t) = [V + \sqrt{2D}\xi(t)]\eta(t)$$



Results and Contributions:

(a) Particle dynamics with fluctuating velocities





Generalizes Lajeunesse et al (2017)

(b) First calculation of flux pdf from particle dynamics

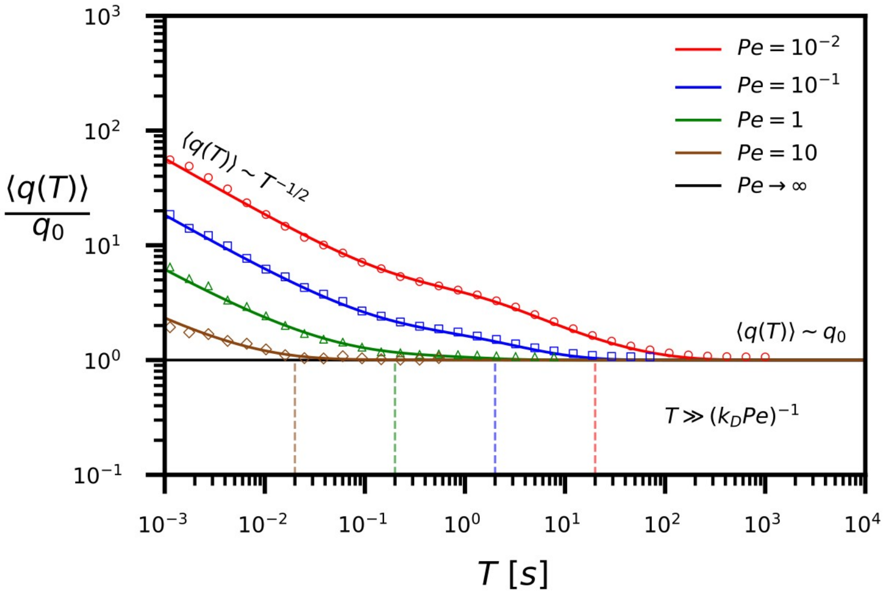
$$P(q|T) = \sum_{l=0}^{\infty} \frac{\Lambda(T)^l}{l!} e^{-\Lambda(T)} \delta(q - \frac{l}{T})$$

Generalizes Ancey et al (2020)

Ch 2. Mechanistic-stochastic formulation of the bedload flux probability distribution

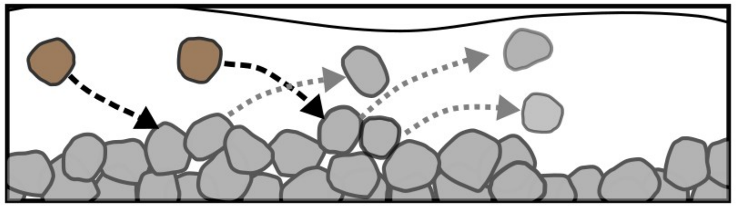
Implications:

(a) Sediment fluxes adopt "scale-dependence"



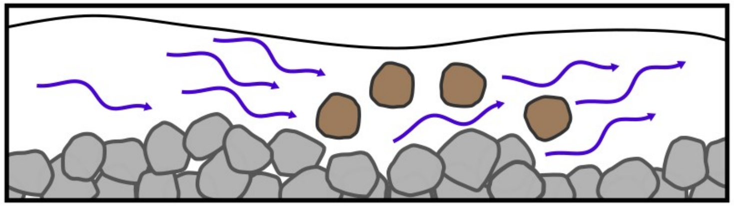
(b) Wide sediment transport fluctuations originate from either

i) Interactions between grains, or



c.f. Chartrand et al (2021)

ii) Correlations in the turbulent flow



Next steps:

(a) Include interactions/correlations

$$\dot{x}_i(t) = [V + \xi_i(t)]\eta_i(t) + \sum_{i \neq j} F_{ij}(x_i, x_j)$$

(b) Evaluate repercussions for landscape evolution

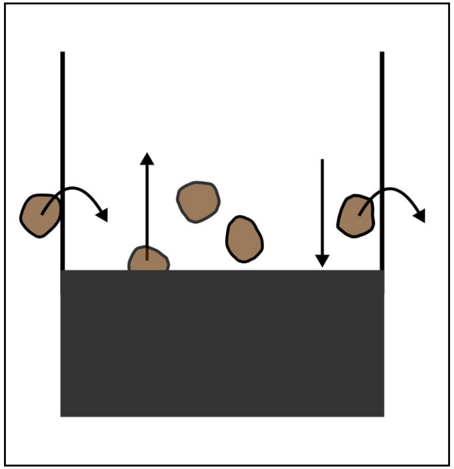
$$(1-\phi)\frac{\partial z}{\partial t}(\mathbf{x}, t) = -\nabla q(\mathbf{x}, t)$$

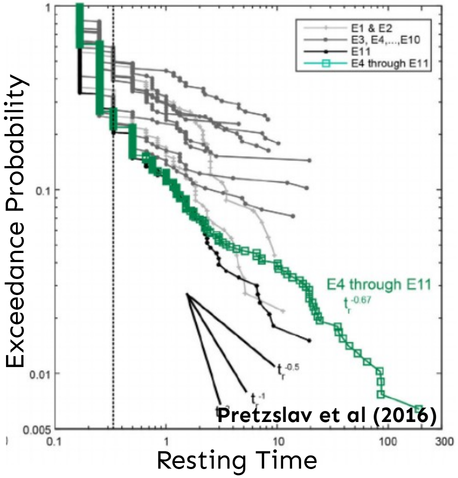
Ch 3. Analysis of bed elevations and sediment transport

<u>Problem statement:</u> [Pierce & Hassan (2020) - JGR]

(a) How bed elevation changes impact sediment transport statistics is unknown

(b) Unclear how to interpret sediment resting times from field measurements



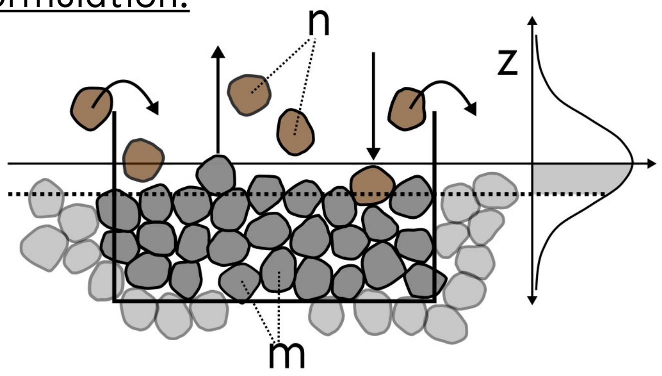


<u>Objectives:</u>

(a) Incorporate bed elevation changes into a sediment transport model,
(b) Evaluate their impact on sediment transport, and
(c) Determine the timescales of sediment burial

Ch 3. Analysis of bed elevations and sediment transport

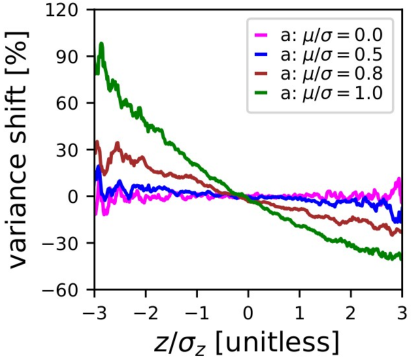
Formulation:



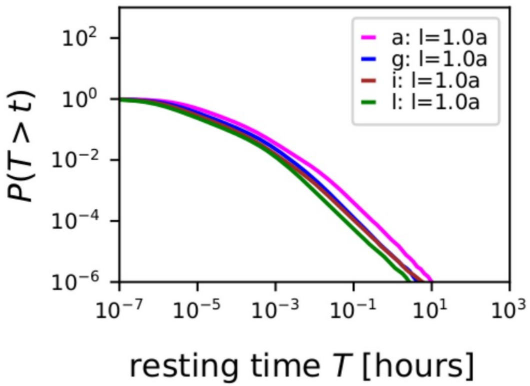
Coupled population model

$$\frac{\partial P}{\partial t}(n,m;t) = \nu P(n-1,m;t) + [\lambda(m+1)+\mu(n-1)][1+\kappa(m+1)]P(n-1,m+1;t)$$
$$+\sigma(n+1)[1-\kappa(m-1)]P(n+1,m-1;t)+\gamma(n+1)P(n+1,m;t)$$
$$-\{\nu+\lambda+\mu n(1+\kappa m)+\sigma n(1-\kappa m)+\gamma n\}P(n,m;t)$$

Results and Contributions:



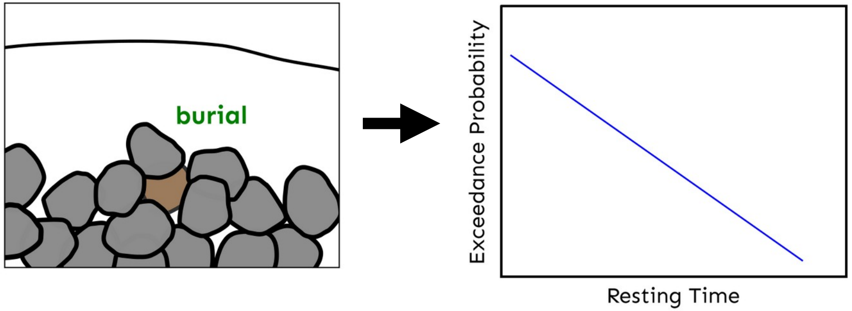
(a) Bed elevations modify sediment transport characteristics



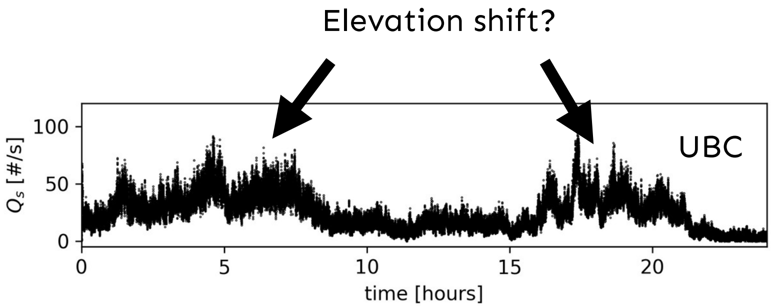
(b) Particles remain buried for power-law distributed timescales

Implications:

(a) Sediment burial can account for the power-law rests seen in field data

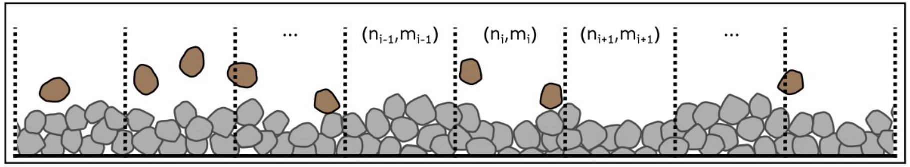


(b) Sediment transport rate distributions are sensitive to local bed elevation adjustments



Next steps:

(a) Stochastic morphodynamics



(b) Sediment dispersal when burial occurs

Ch 4. Burial-induced three-range diffusion in bedload sediment transport

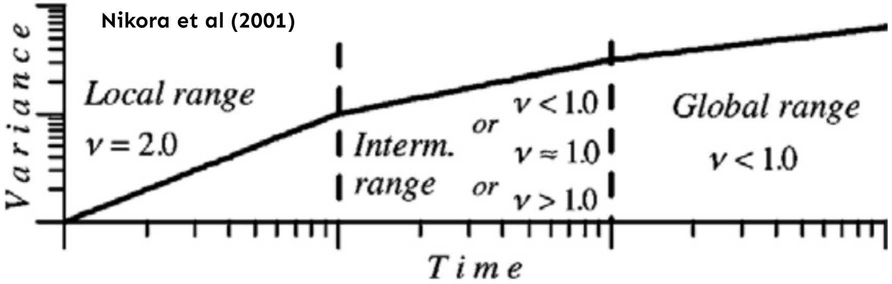
Problem statement:

[Pierce & Hassan (2020) - GRL]

(a) Tracer particles progressively settle into a uniform distribution

Bradley et al (2012)

(b) Grains spread out at three distinct rates depending on observation time



No precise explanation yet for either of these observations

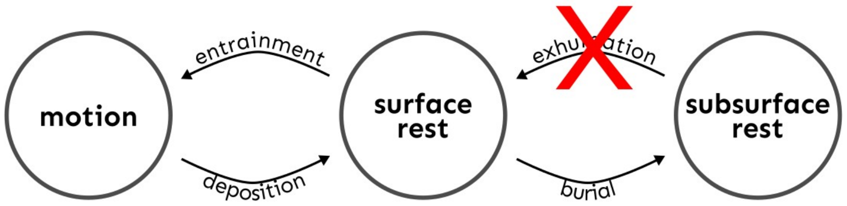
Objectives:

(a) Formulate the downstream movements of particles with motion/rest/burial

(b) Explain uniformity & three-range spreading characteristics

Ch 4. Burial-induced three-range diffusion in bedload sediment transport

Formulation:



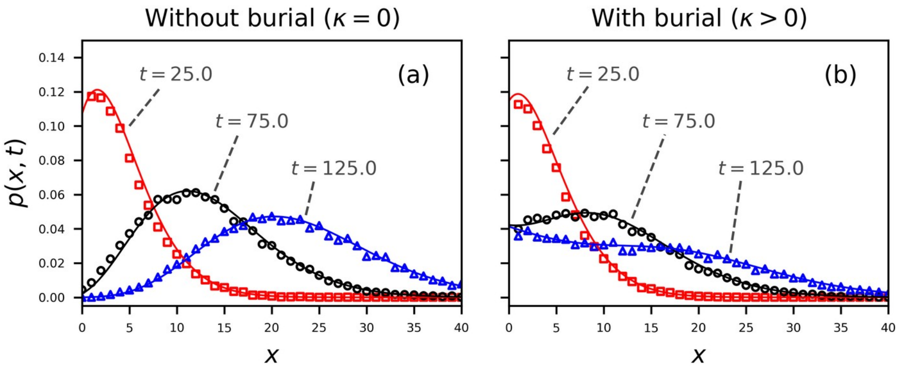
$$\omega_{ij}(x,\, t) = \theta_i g_{ij}(x,\, t) + \sum_{k=0}^{2} \int_0^x dx' \int_0^t dt' \omega_{ki}(x',\, t') g_{ij}(x - x',\, t - t')$$

$$p_i(x,\, t) = \theta_i G_i(x,\, t) + \sum_{k=0}^{2} \int_0^x dx' \int_0^t dt' \omega_{ki}(x',\, t') G_i(x - x',\, t - t')$$

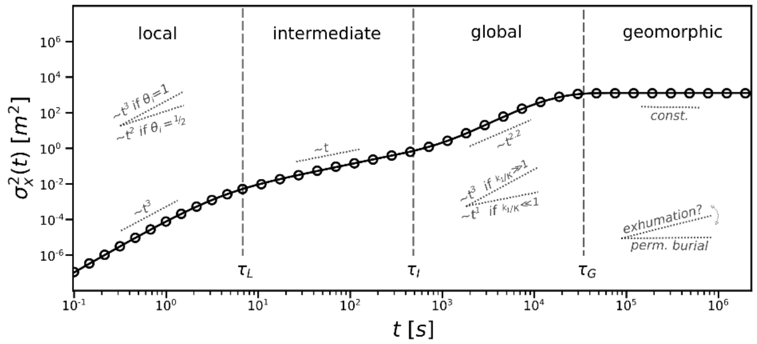
Application of multi-state random walk formalism from Weiss (1994)

Results & contributions:

(a) Derivation of uniform distribution tendency



(b) First description of "three-range diffusion"



Ch 4. Burial-induced three-range diffusion in bedload sediment transport

Implications:

(a) Contaminant transport

(b) Understanding tracer movement

Papangelakis (2015)

Next steps:

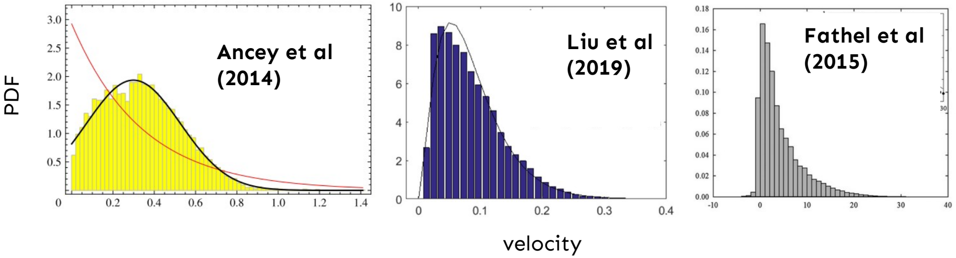
(a) Incorporate the exhumation process
Using the timescales derived in Ch. 2

(b) Apply the formalism to back-calculate
sediment transport rates from tracer data

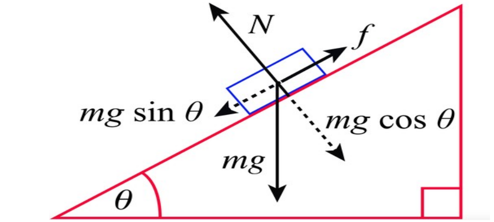
Ch 5. Collisional Langevin description of bedload velocity distributions

Problem statement:

(a) Bedload velocity characteristics lack any comprehensive explanation



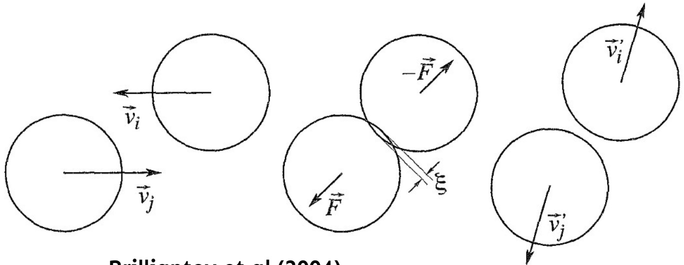
(b) Existing models account for collisions in an unrealistic way



Objectives:

(a) Formulate grain-scale transport with realistic collision model

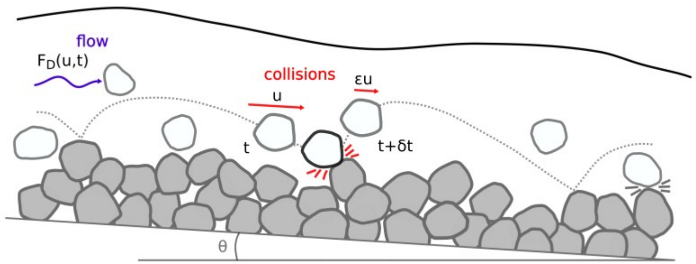
(b) Explain all earlier bedload velocity observations

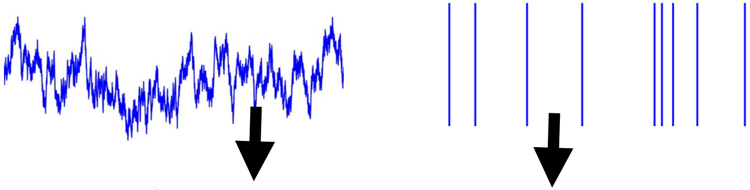


Brilliantov et al (2004)

Ch 5. Collisional Langevin description of bedload velocity distributions

Formulation:



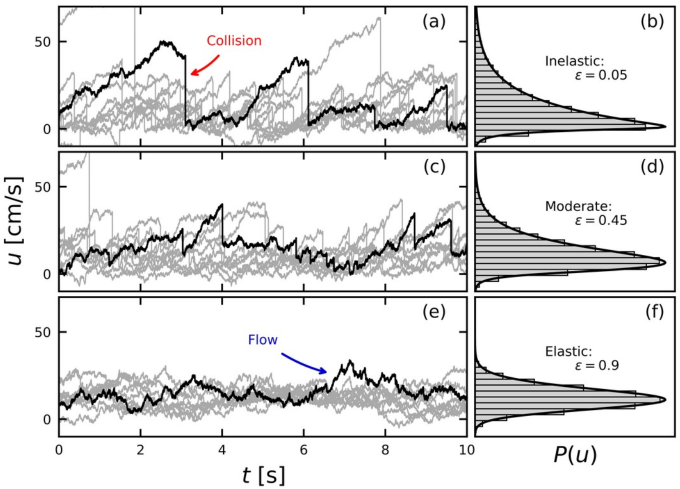


$$m\dot{u}(t) = \Gamma + \sqrt{2D}\eta(t) - mu(t)\xi_{\nu,\varepsilon}(t)$$

Turbulence | Collisions

Results and Contributions:

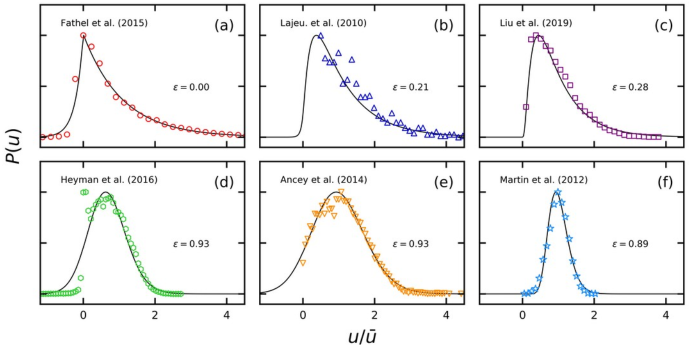
(a) Collisions control velocity pdf shape



(b) Sediment transport is a rarefied granular gas

$$\nu^{-1}\partial_t P(u,t) = -\Gamma\partial_u P(u,t) + D\partial_u^2 P(u,t) - P(u,t) + \int_0^1 \frac{d\varepsilon}{\varepsilon} P\left(\frac{u}{\varepsilon}, t\right)\rho(\varepsilon)$$

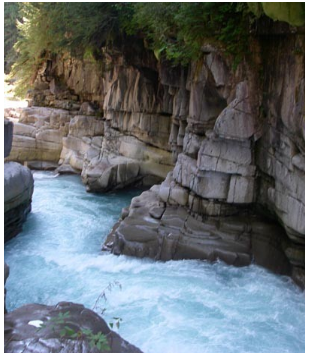
(c) Unified explanation of experiments



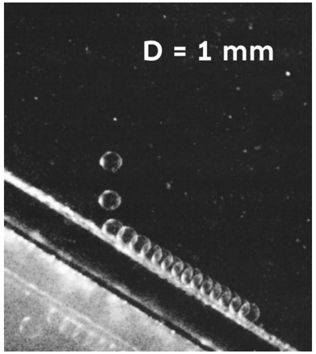
Ch 5. Collisional Langevin description of bedload velocity distributions

Implications:

(a) Bedrock canyon evolution

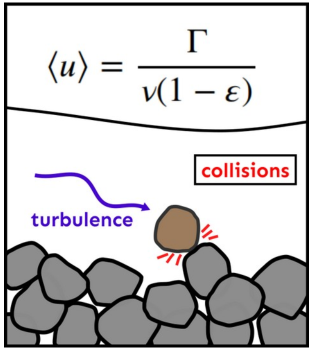


(b) Particle size dependence



Schmeeckle et al (2001)

(c) Sensitivity to collisions, not turbulence

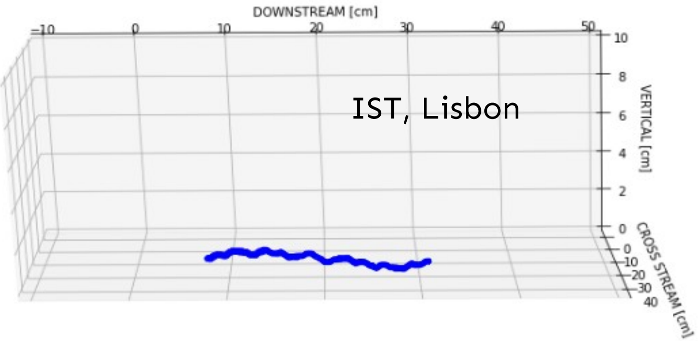


Next steps:

(a) Formulate in 3D

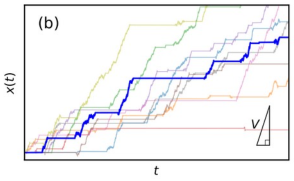
$$m\dot{\boldsymbol{u}}(t) = \mathbf{F} + D\boldsymbol{\xi}(t) - m\boldsymbol{u} \otimes \boldsymbol{\eta}$$

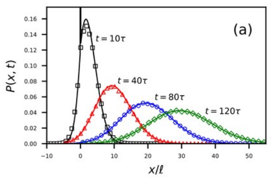
(b) Compare with experiments

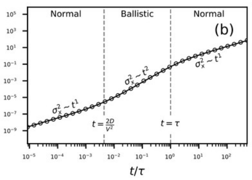


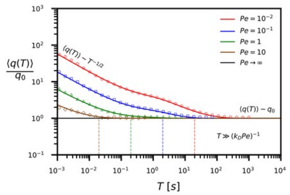
Ch 6. Summary of contributions

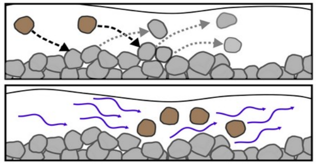
Ch 2. Mechanistic-stochastic formulation of the bedload flux probability distribution



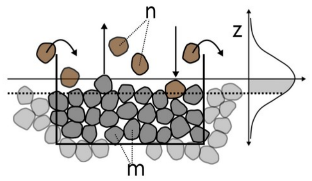


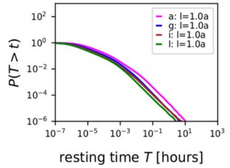




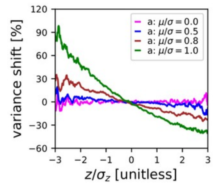


Ch 3. Analysis of bed elevation change and sediment transport fluctuations

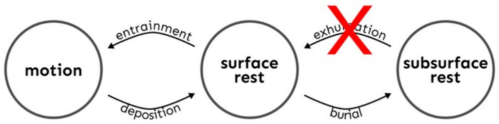


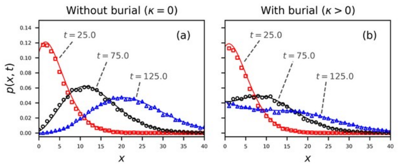


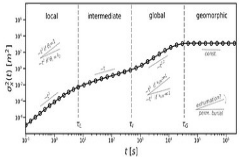
$$\frac{\partial P}{\partial t}(n,m;t) = \nu P(n-1,m;t) + [\lambda(m+1)+\mu(n-1)][1+\kappa(m+1)]P(n-1,m+1;t)$$
$$+\sigma(n+1)[1-\kappa(m-1)]P(n+1,m-1;t) + \gamma(n+1)P(n+1,m;t)$$
$$-\{\nu+\lambda+\mu n(1+\kappa m)+\sigma n(1-\kappa m)+\gamma n\}P(n,m;t)$$



Ch 4. Burial-induced three-range diffusion in sediment transport

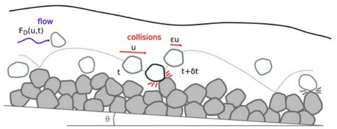


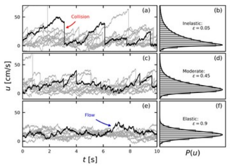


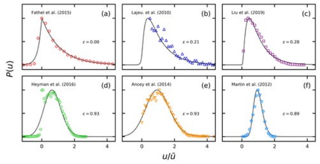


$$\omega_{ij}(x,\,t) = \theta_i g_{ij}(x,\,t) + \sum_{k=0}^{2}\int_0^x dx' \int_0^t dt' \omega_{ki}(x',\,t') g_{ij}(x-x',\,t-t')$$
$$p_i(x,\,t) = \theta_i G_i(x,\,t) + \sum_{k=0}^{2}\int_0^x dx' \int_0^t dt' \omega_{ki}(x',\,t') G_i(x-x',\,t-t')$$

Ch 5. Collisional Langevin description of bedload velocity distributions





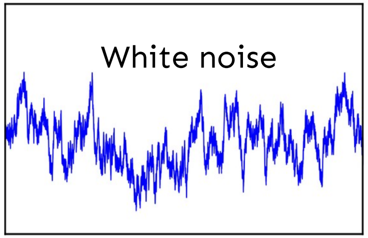


$$P(u) = \frac{\theta(-u)}{K_+}\sum_{l=0}^{\infty}\frac{\varepsilon^{-l}e^{\lambda_+\varepsilon^{-l}u}}{\prod_{m=1}^{l}(-\tilde{D}\lambda_+^2\varepsilon^{-2m}+\tilde{\Gamma}\lambda_+\varepsilon^{-m}+1)} + \frac{\theta(u)}{K_-}\sum_{l=0}^{\infty}\frac{\varepsilon^{-l}e^{\lambda_-\varepsilon^{-l}u}}{\prod_{m=1}^{l}(-\tilde{D}\lambda_-^2\varepsilon^{-2m}+\tilde{\Gamma}\lambda_-\varepsilon^{-m}+1)}$$

Ch 6. The stochastic methodology in Earth science

"The development of land forms by erosional and gradational processes still remains largely qualitative. (...) This is probably the result largely of lack of adequate tools with which to work, and these tools must be of two kinds: measuring tools and **operating tools**" (Horton, 1945).

Idealized noises → Langevin models → Master equations → Probabilistic descriptions



$$\dot{u}(t) = -\Delta \mathrm{sgn}(u) + F + \sqrt{2D}\xi(t)$$

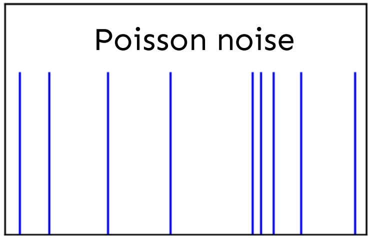
$$\frac{\partial}{\partial t}P(u,t) = -\Delta\frac{\partial}{\partial u}\Big[\mathrm{sgn}(u)P\Big] + D\frac{\partial^2 P}{\partial u^2}$$

$$P(u) = \frac{\Delta^2 - F^2}{2\Delta D}\exp\Big(-\frac{-\Delta|u| + Fu}{D}\Big).$$

$$m\dot{u}(t) = \Gamma + \sqrt{2D}\eta(t) - mu(t)\xi_{\nu,\varepsilon}(t)$$

$$\nu^{-1}\partial_t P(u,t) = -\Gamma\partial_u P(u,t) + D\partial_u^2 P(u,t) - P(u,t) + \int_0^1 \frac{d\varepsilon}{\varepsilon}P\Big(\frac{u}{\varepsilon},t\Big)\rho(\varepsilon)$$

$$P(u) = \frac{\theta(-u)}{K_+}\sum_{l=0}^{\infty}\frac{\varepsilon^{-l}e^{\lambda_+\varepsilon^{-l}u}}{\prod_{m=1}^{l}(-\tilde{D}\lambda_+^2\varepsilon^{-2m} + \tilde{\Gamma}\lambda_+\varepsilon^{-m} + 1)} + \frac{\theta(u)}{K_-}\sum_{l=0}^{\infty}\frac{\varepsilon^{-l}e^{\lambda_-\varepsilon^{-l}u}}{\prod_{m=1}^{l}(-\tilde{D}\lambda_-^2\varepsilon^{-2m} + \tilde{\Gamma}\lambda_-\varepsilon^{-m} + 1)}$$



$$t_r\dot{u}(t) = -(U-u) + \sqrt{2D}\xi(t).$$

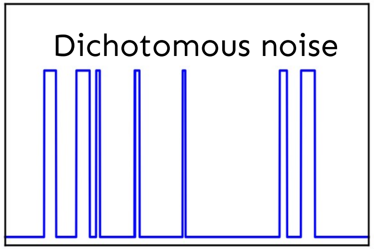
$$\frac{\partial}{\partial t}P(u,t) = -\frac{\partial}{\partial u}\Big[\frac{U-u}{t_r}P\Big] + \frac{D}{t_r^2}\frac{\partial^2 P}{\partial u^2}$$

$$P(u) = \sqrt{\frac{t_r}{2\pi D}}\exp\Big(-\frac{t_r(u-U)^2}{2D}\Big)$$

$$\dot{x} = V\eta(t)$$

$$\big(\partial_t^2 + V\partial_x\partial_t + k_E V\partial_x + k\partial_t\big)P(x,t) = 0$$

$$P(x,t) = e^{-\chi-\tau}\Big[\frac{k_E}{V}\delta(\tau) + \frac{k_E}{V}\sqrt{\frac{\chi}{\tau}}\mathcal{I}_1(2\sqrt{\chi\tau}) + \frac{k_D}{V}\mathcal{I}_0(2\sqrt{\chi\tau}) + \frac{k_E k_D}{kV}\sqrt{\frac{\tau}{\chi}}\mathcal{I}_1(2\sqrt{\chi\tau}) + \frac{k_E k_D}{kV}\delta(\chi)\Big]\theta(\chi)\theta(\tau)$$



$$\dot{x}(t) = \mu(t).$$

$$\big(\ell\partial_x\partial_t + k_E\ell\partial_x + \partial_t\big)P(x,t) = 0.$$

$$P(x,t) = \Big[\delta(x)e^{-k_E t} + e^{-k_E t - x/\ell}\sqrt{\frac{k_E t}{\ell x}}\mathcal{I}_1\Big(2\sqrt{\frac{k_E x t}{\ell}}\Big)\Big]\theta(x)\theta(t)$$

$$\dot{x}(t) = [V + \sqrt{2D}\xi(t)]\eta(t)$$

$$\partial_t^2 P + V\partial_x\partial_t P + k_E V\partial_x P + k\partial_t P - D\partial_x^2\partial_t P - k_E D\partial_x^2 P = 0$$

$$P(x,t) = \big[-\varphi D\partial_x^2 + V\varphi\partial_x + k + \delta(t) + \partial_t\big] \times \int_0^t \mathcal{I}_0\Big(2\sqrt{k_E k_D u(t-u)}\Big)e^{-k_E(t-u)-k_D u} \times \sqrt{\frac{1}{4\pi D u}}\exp\Big[-\frac{(x-Vu)^2}{4Du}\Big]du$$

Variability is ok !

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