A statistical mechanics of rarefied sediment transport

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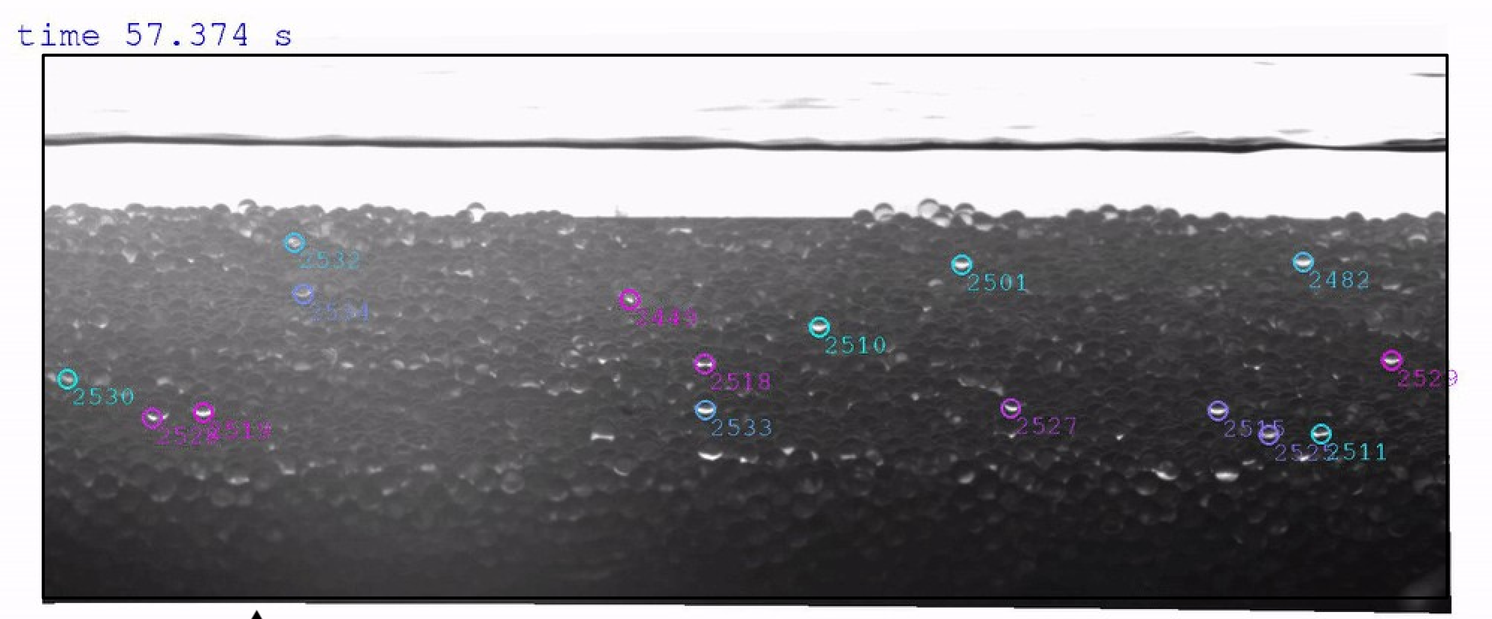
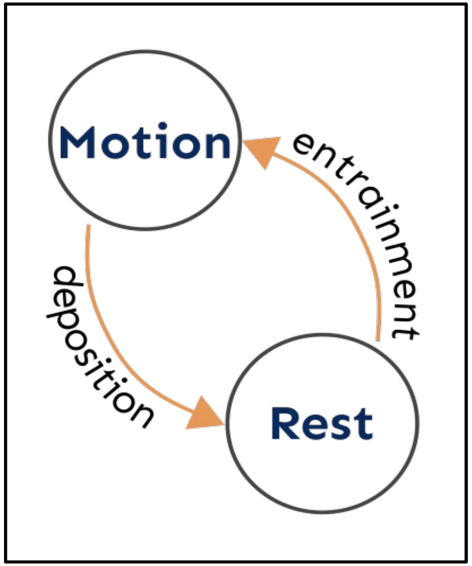
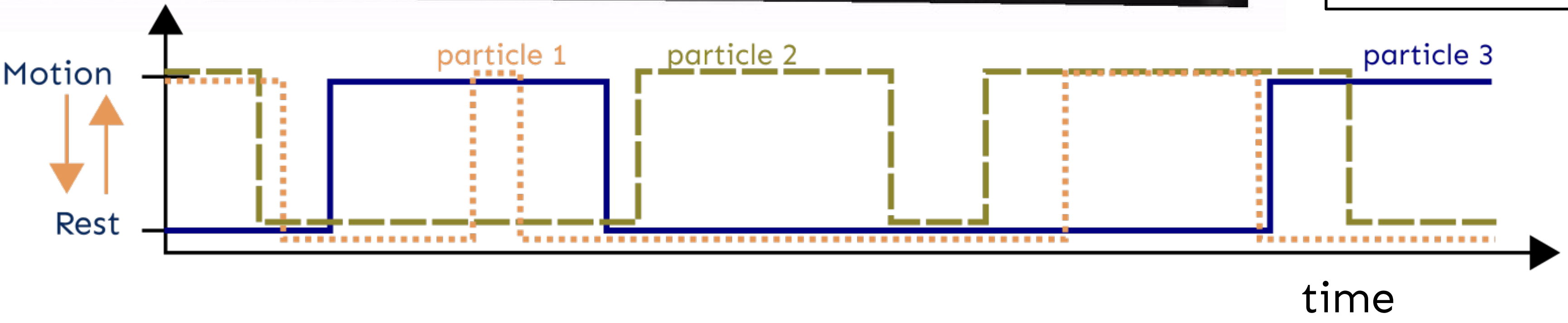
Rarefied bedload transport





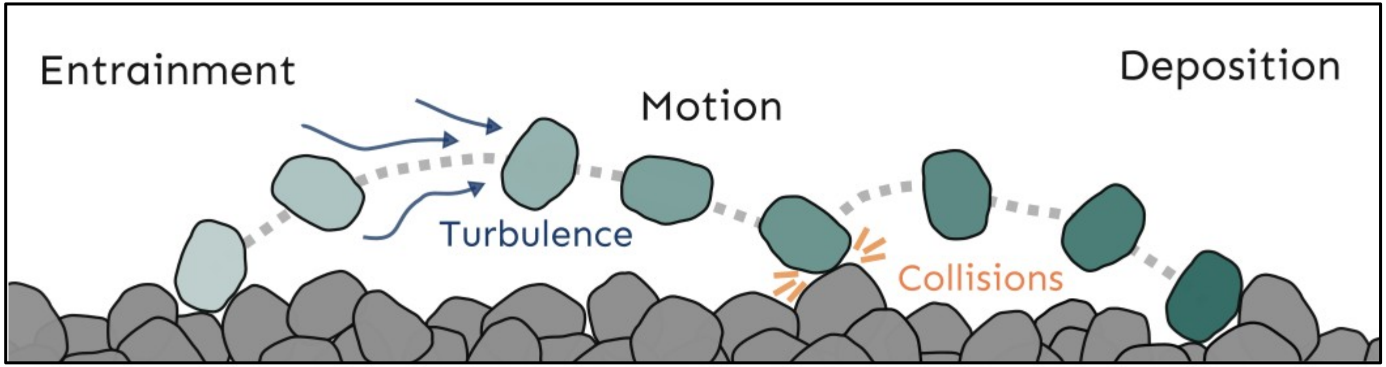


Rarefied means that collisions among moving particles are uncommon

Current understanding of rarefied transport

Key problems:

(1) Transitions; (2) Velocities; (3) Displacements; (4) Fluxes



Major limitation: Models remain largely phenomenological

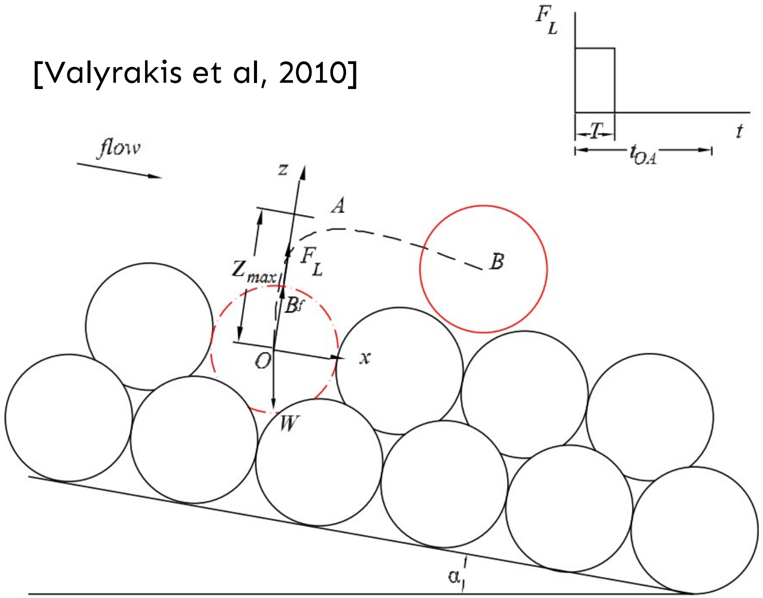
"**Mechanistic models are more powerful** since they tell you about the **underlying processes driving patterns** and allow for **extrapolation beyond the observed conditions**." [Bolker, 2008]

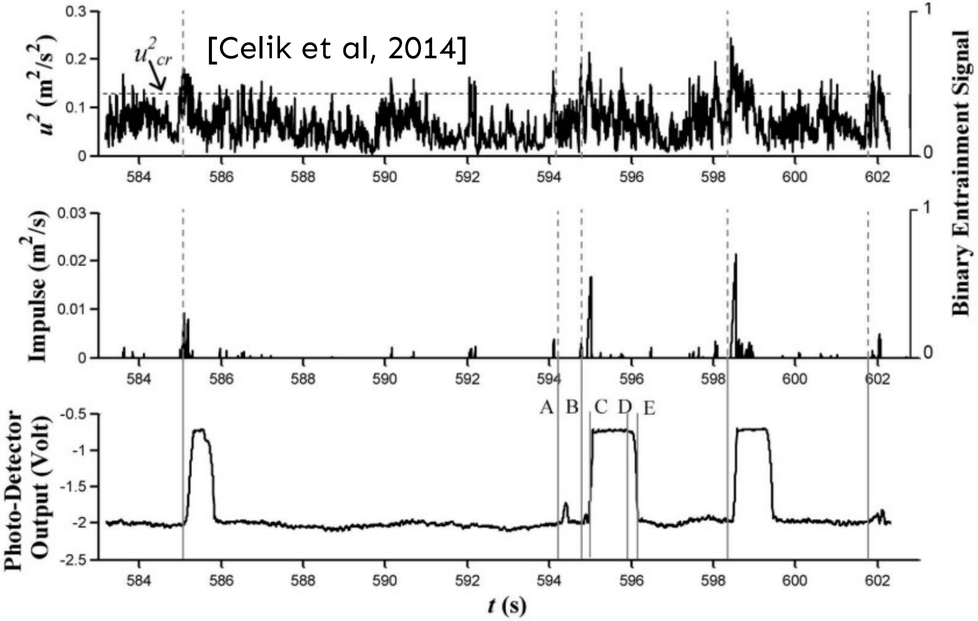
Objective: formulate rarefied transport **mechanistically**

Particles entrain due to **fluid impulse** [Diplas et al, 2008; Celik et al, 2014; etc]

Impulse:

Particles can shuttle around without leaving their pockets





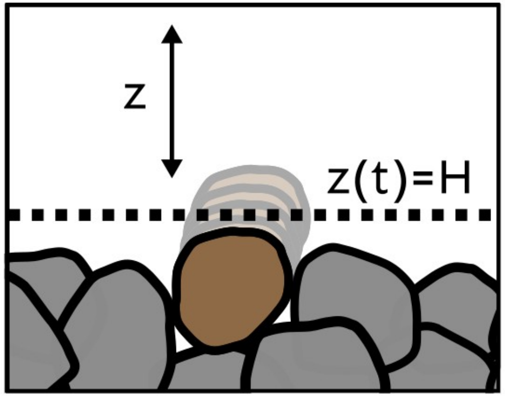
Existing impulse models provide binary (yes/no) condition, not timescales

Entrainment as a first passage problem

Pocket shuttling: Lift vs weight for demonstration [cf. Einstein, 1950]

$$m\ddot{z}(t) = -\gamma \dot{z} - m_* g + F_L(t)$$

Inertia Stokes Weight Turbulent Lift



First passage time:

Exponential resting time distribution

$$f_R(t) = k_E \exp(-k_E t)$$

Entrainment rate

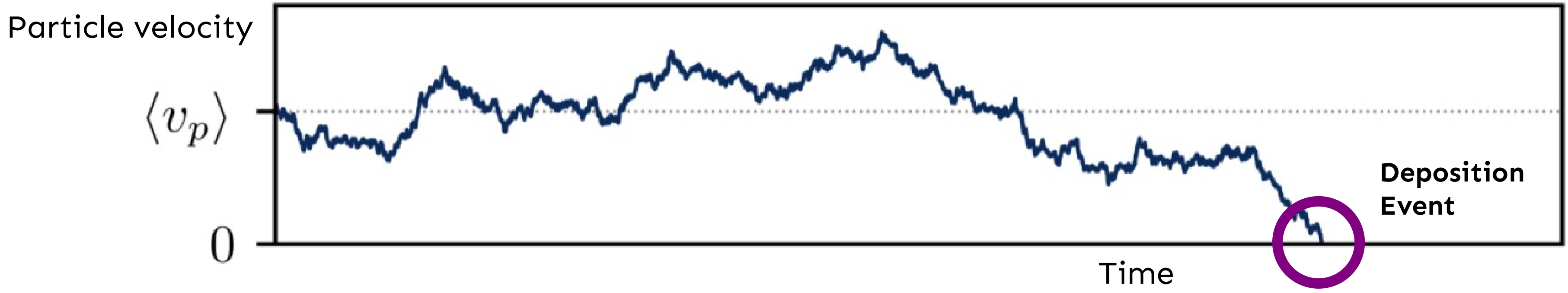
$$k_E \approx \frac{\langle F_L^2 \rangle \tau_c}{(\gamma H)^2}$$

Impulse produces exponentially-distributed resting times

Deposition requires pocket encounter at sufficiently low velocity

Simplification:

A first approximation ignores bed topography – velocity only



First passage time:

Movement time distribution

$$f_M(t) = k_D \exp(-k_D t)$$

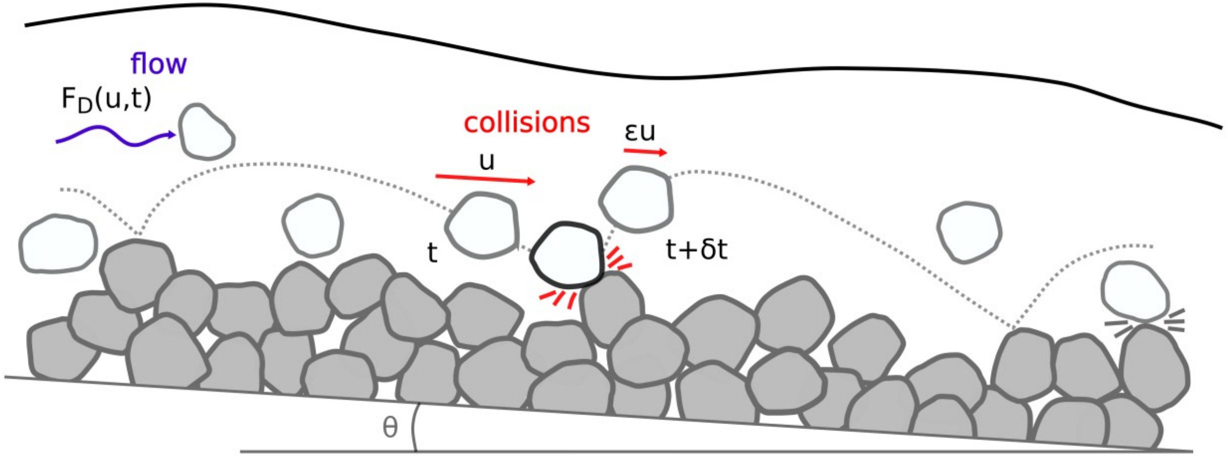
Deposition rate

$$k_D \approx \frac{\sqrt{\gamma \tau_c \langle F_D^2 \rangle / \pi}}{m \langle v_p \rangle}$$

First hitting model implies exponentially-distributed movement times

Newtonian descriptions of bedload velocities

Key components:



1. Turbulent forcing
2. Collisions
   (rate, elasticity)

Earlier velocity models:

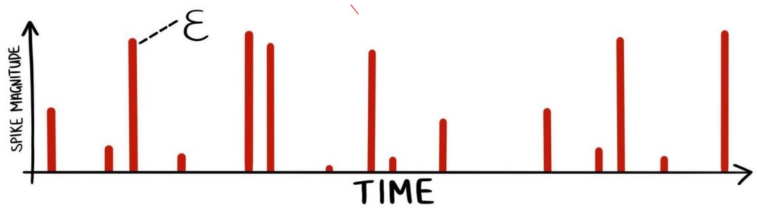
Based on effective friction terms – no collisions

$$\dot{v}(t) = F(v) + \xi(t)$$

[Ancey et al, 2014; Fan et al, 2014, Furbish, et al 2021]

Existing models describe only exponential or Gaussian velocities

Improved episodic collision model for particle velocities



Episodic Langevin model:

$$m\dot{v}(t) = F(v,t) - mv \sum_{n=0}^{N_\nu(t)} (1-\varepsilon_n)\delta(t-\tau_k)$$

Boltzmann-like description:

$$\partial_t P(u,t) = -\frac{\langle F\rangle}{m}\partial_u P + \frac{\sigma_F^2 \tau_c}{2m^2}\partial_u^2 P + \nu \int_0^1 \frac{d\varepsilon}{\varepsilon} P\left(\frac{u}{\varepsilon},t\right)\Big[\rho(\varepsilon) - \delta(\varepsilon-1)\Big]$$

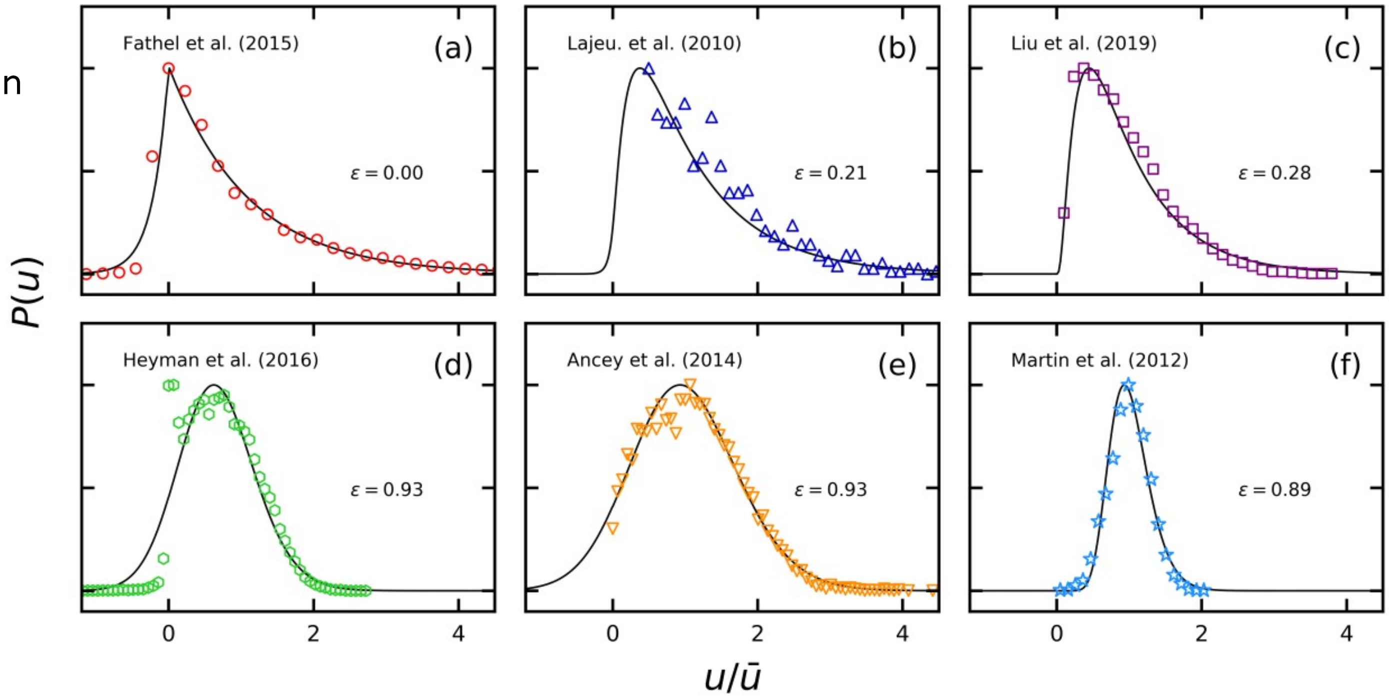
Bedload velocities obey equations analogous to granular gases

Comparison to experimental data

Velocity distributions:

Collision rate vs momentum dissipated per collision

Low rate
High dissipation

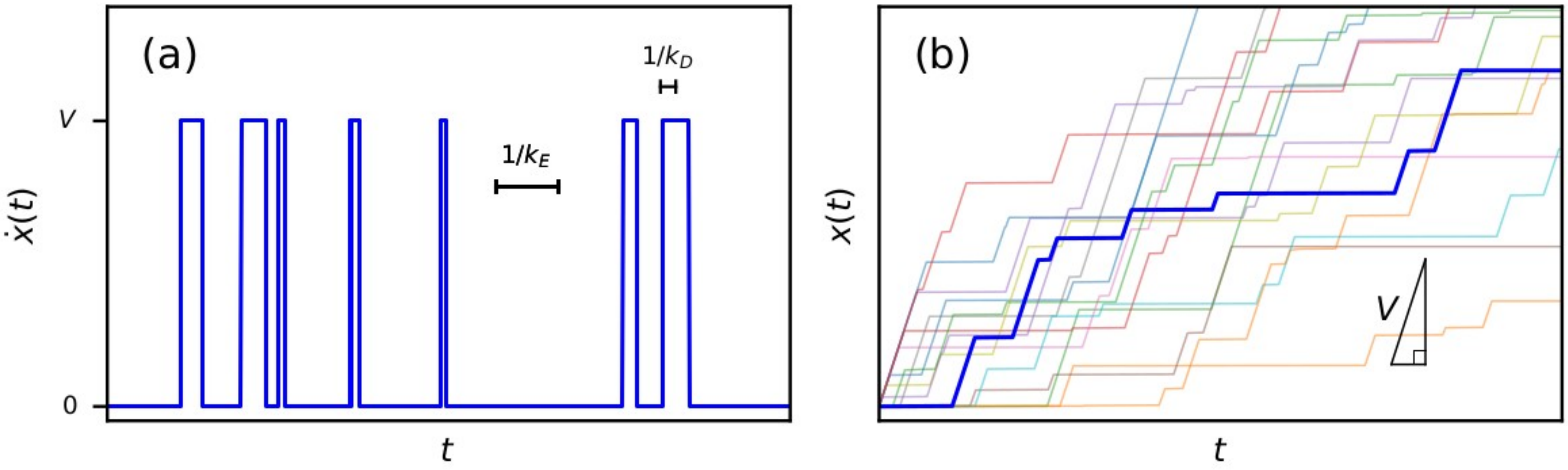


High rate
Low dissipation

Collision characteristics explain the range of velocity observations

Earlier models of displacement via motion-rest alternation

Earlier "random walk" descriptions: [Pelosi et al, 2016; Lajeunesse et al, 2019; Pierce et al, 2020; etc]



A new way to formulate the same models ...

$$\dot{x} = V\eta(t)$$

A stochastic "on-off" switch

Random walk models oversimplify the short timescale dynamics

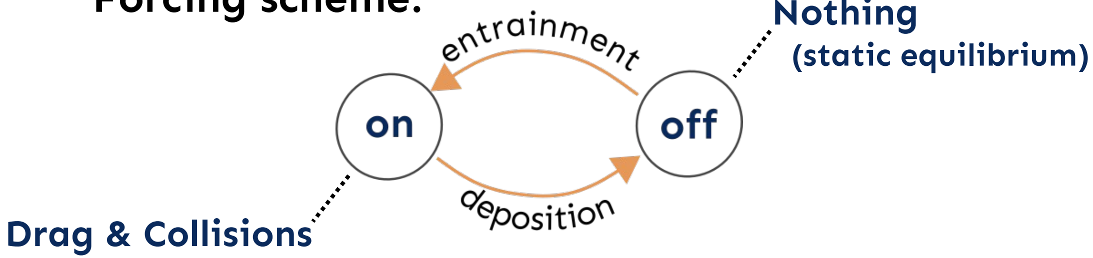
Displacement via motion-rest alternation

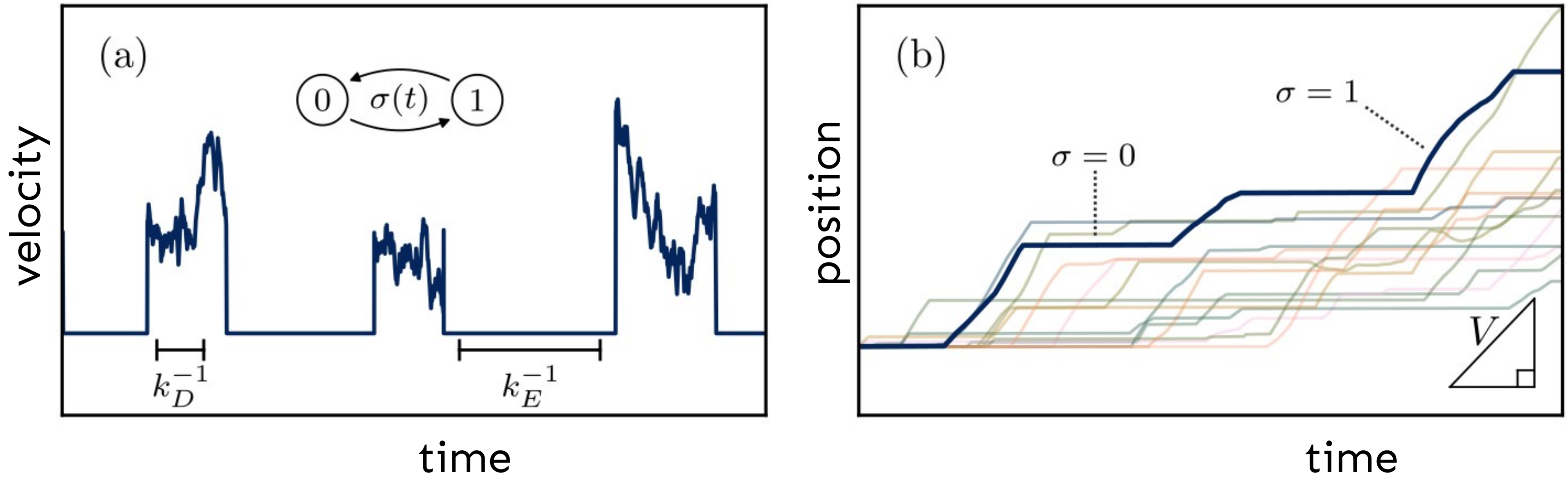
Stochastic description:

$$\dot{x}(t) = v(t)\sigma(t)$$

$$\dot{v}(t) = [F(v) + \xi(t)]\sigma(t)$$

Forcing scheme:





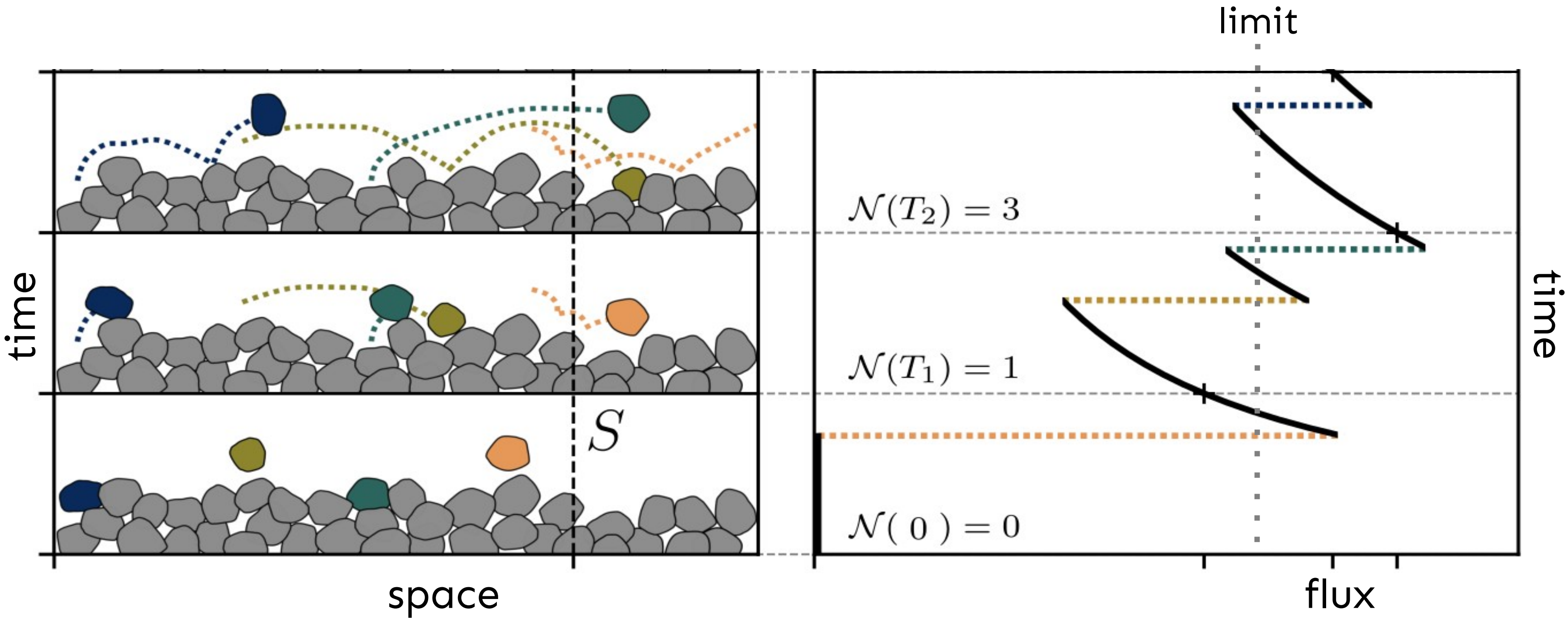
Mechanistic description outperforms random walk models at short timescales

The flux as the number of particles crossing a surface in T

Definition:

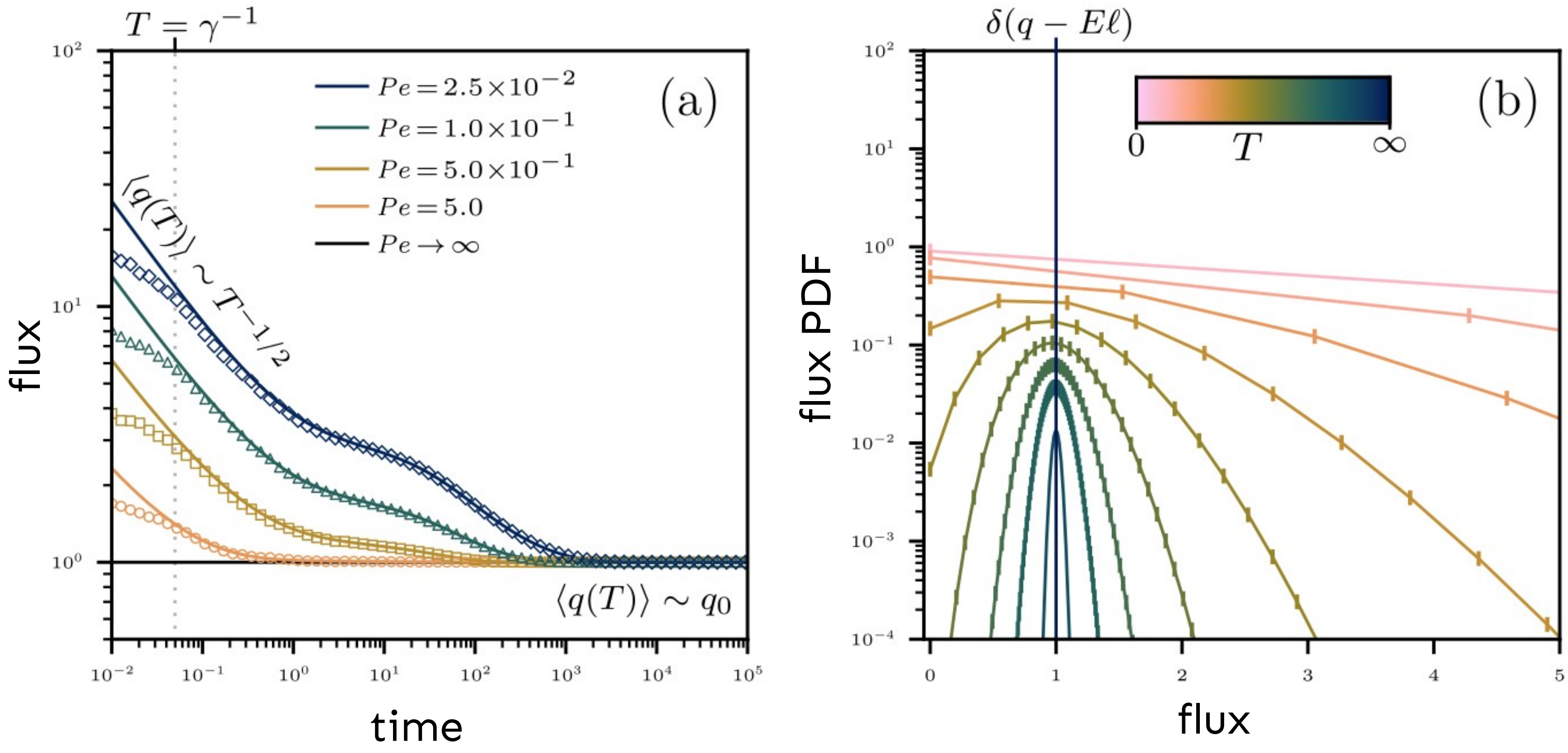
$$q(T) = \frac{\mathcal{N}(T)}{T}$$

Particle dynamics



Transport rates fluctuate and depend on the observation time

The flux distribution depends on the observation time



Transport rates are ambiguous without additional info about observation times

Summary

Developed stat mech description for all components of rarefied bedload transport

(1) Transitions, (2) Velocities, (3) Displacements, (4) Fluxes

Some Implications:

1. First passage approach derives motion/rest timescales
2. Collisions control bedload velocity distributions
3. Sediment fluxes are incomplete without observation times

Some next steps:

1. Rest and motion timescale experiments (in progress)
2. Deposition modeling with bed geometry (in progress)
3. Computational physics & relation to stochastics (in progress)
4. Particle-particle interactions & collective effects (soon)

Thanks!

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Related material:

Pierce et al, 2020 – JGR:ES
Pierce et al, 2020 – GRL
Pierce, 2021 – PhD Thesis
Pierce et al, 2022 – In review @ ESurf